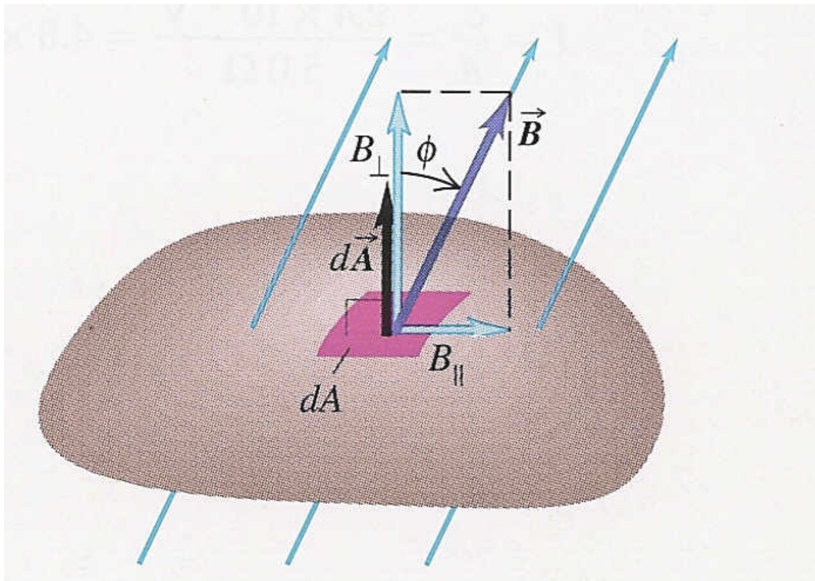
Electromagnetic Induction

Key Concept is Magnetic Flux



MAGNETIC FLUX

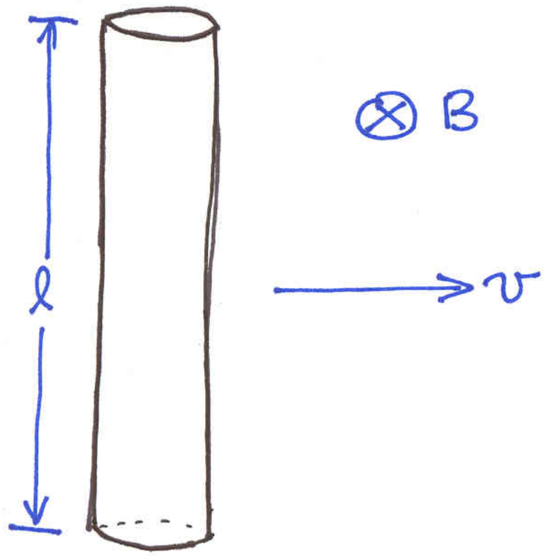
$$d\Phi_B = \vec{B}\cdot d\vec{A} = B_\perp dA = B\, dA \cos\phi$$

TOTAL FLUX

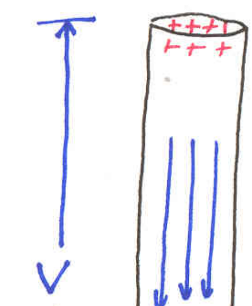
$$\Phi_B = \int_{\text{Surface}} \vec{B}\cdot d\vec{A} = \int_{\text{Surface}} B\, dA \cos\phi$$

If $\vec{B}$ is constant and A is FLAT

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi$$



The charges in the conductor will feel a magnetic force given by $\vec{F} = q\,\vec{v} \times \vec{B}$ and will move to the ends of the cylinder –



– This displacement of charge will give rise to a potential difference between the top and the bottom

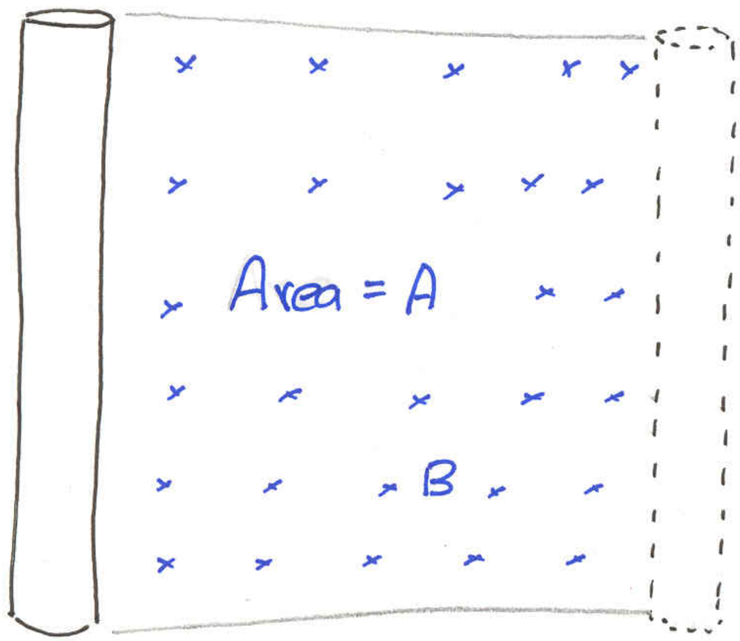
– Charge will continue to move until the magnetic Force $F = q\,v \times B$ is just balanced by the electric force $F = qE \rightarrow$ $\boxed{V = lE}$

$$qvB = qE$$

$$vB = V/l$$

$$V = lvB$$

$$\ell v = \frac{dA}{dt}$$



$$V = \frac{d}{dt}(BA)$$

Recall BA is a magnetic flux

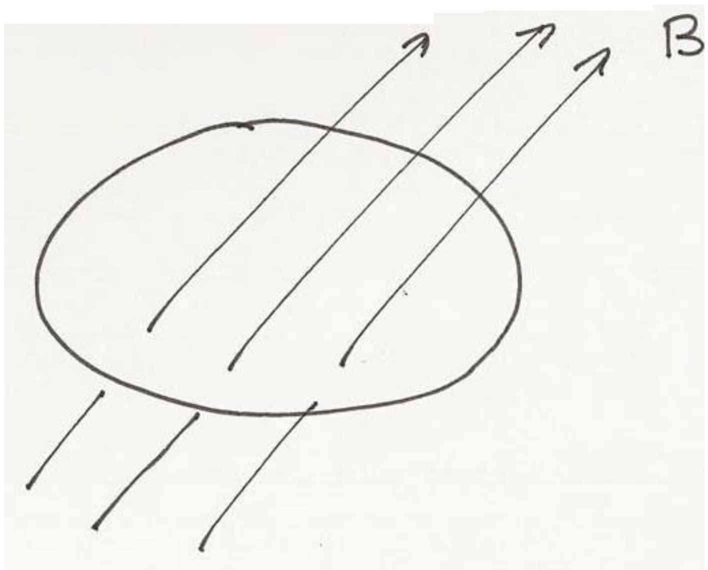
$$BA = \int_{\text{surface}} B \cdot dA = \Phi_B$$

So $\longrightarrow$

$$\boxed{|V| = \frac{d}{dt}\Phi_B}$$

<u>Faraday's Law</u>

Key Concept is **CHANGE** in Magnetic Flux



A CHANGING Φ_B through any closed loop induces" an EMF around the loop.

The induced EMF equals the negative of the time rate of change of the total magnetic flux through the loop

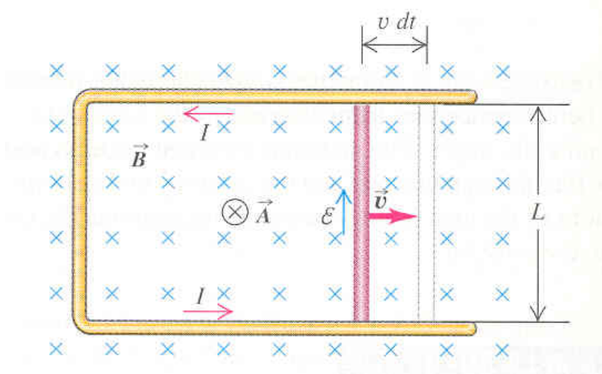
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's Law

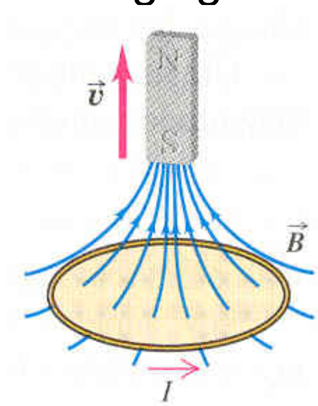
$$\varepsilon = -\frac{d\Phi_B}{dt} \quad where \quad \Phi_B = \int_{surface} \vec{B}\cdot d\vec{A} = \int_{surface} BdA\cos\phi$$

It doesn't matter why the flux changes

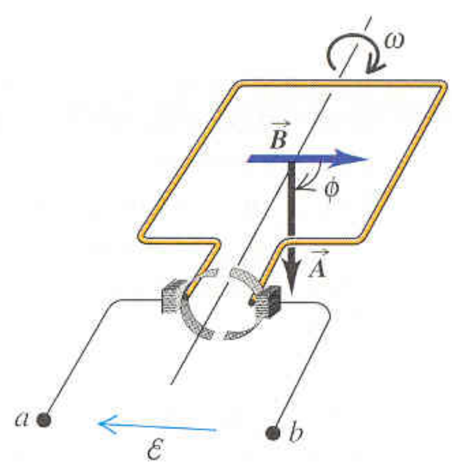
1) Constant B, Changing Area:



2) Constant Area, Changing B:



3) Constant Area, Constant B, Changing Cos φ:



30–18 The conducting rod *ab* in Fig. 30–26 makes contact with metal rails *ca* and *db*. The apparatus is in a uniform magnetic field 0.800 T, perpendicular to the plane of the figure. a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s. ~~b) In what direction does the current flow in the rod?~~ c) If the resistance of the circuit *abdc* is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. Neglect friction. d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

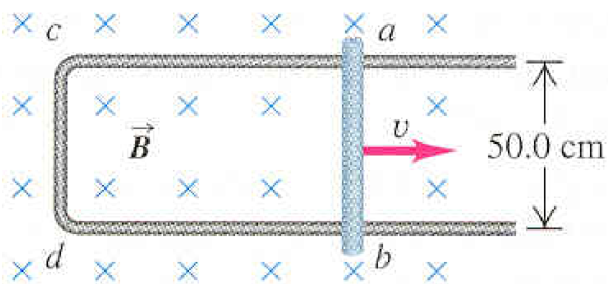


FIGURE 30–26 Exercise 30–18.

a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -B\dfrac{dA}{dt} = -Bv\ell$

$\mathcal{E} = -(0.8T)(7.5\,m/s)(0.5\,m) = \boxed{-3V}$

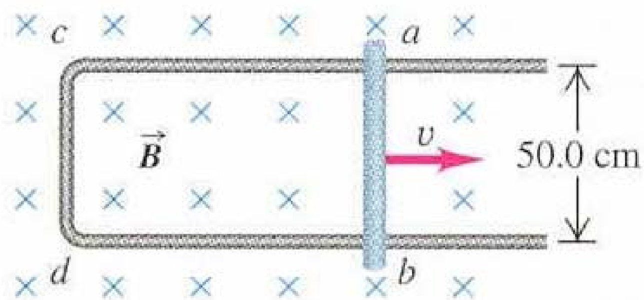
c) $V = IR \Rightarrow I = \dfrac{\mathcal{E}}{R} = \dfrac{3V}{1.5\Omega} = 2A$

$F_{magnetic} = BI\ell = (0.8T)(2A)(0.5\,m) = \boxed{0.8N}$

d) Mechanical Power $= Fv = (0.8N)(7.5\,m/s) = \boxed{6W}$

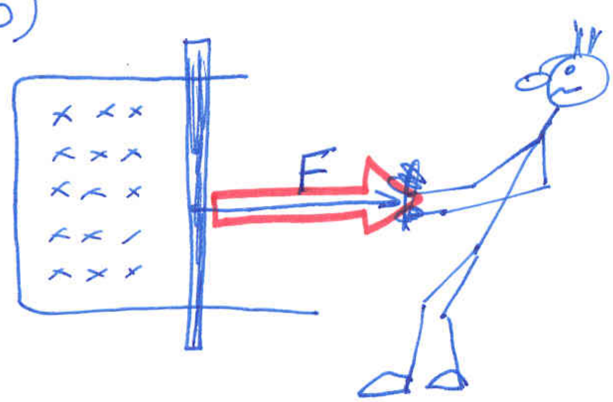
Electrical Power $= \dfrac{V^2}{R} = \dfrac{(3V)^2}{1.5\Omega} = \dfrac{9V^2}{1.5\Omega} = \boxed{6W}$

toward the right with a speed 7.50 m/s. b) In what direction does the current flow in the rod? c) If the resistance of the circuit *abdc* is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. Neglect friction. d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

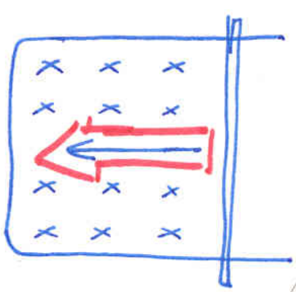


I am doing work and am applying a Force. The slider is <u>NOT</u> accelerating So there must be an equal and opposite force on it in the opposite direction

b)



I must pull in This direction, therfore the magnetic force must be in the opposite direction



$\vec{F} = I\vec{l} \times \vec{B}$

B is into paper
$\vec{F}$ is to the left

Current is up I

Direction of the Induced EMF's and Currents

In the previous problem, we found the direction of the induced current by noting that the force resulting from the induced current had to oppose the applied force. This obbservation can be generalized into:

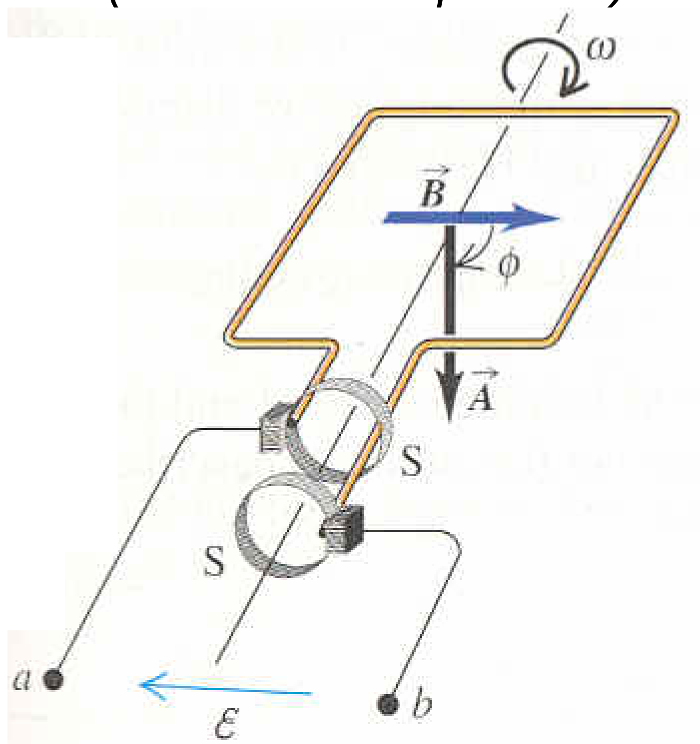
Lenz's Law

The direction of any magnetic induction effect is such as to oppose the cause of the effect

Note

Lenz's Law is an example of the general principle that there is no such thing as a free lunch (otherwise known as the first law of thermodynamics)

The "Alternator"
(see Y&F example 30-4)

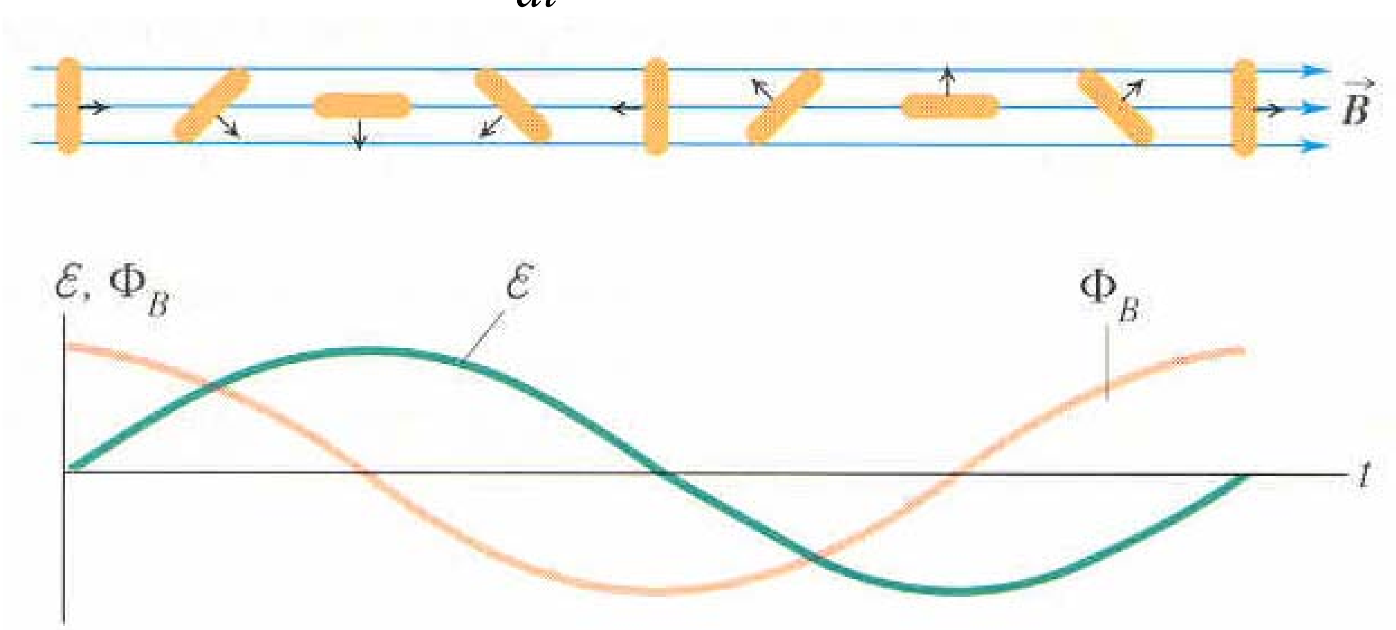


The magnetic field $\vec{B}$ *,and the angular frequency* ω*, are constant*

$$\Phi_B = BA\cos\phi = BA\cos\omega t$$

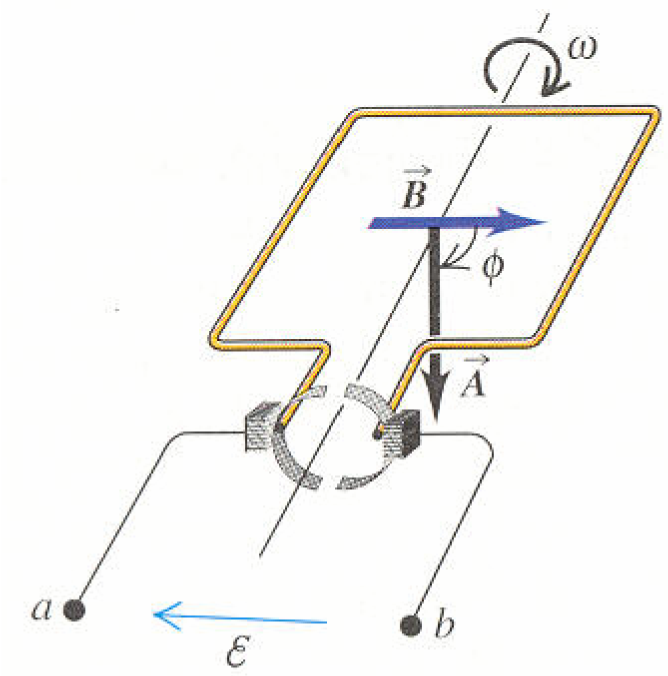
By Faraday's Law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = \omega AB\cos\omega t$$



The "Generator"

(see Y&F example 30-5)

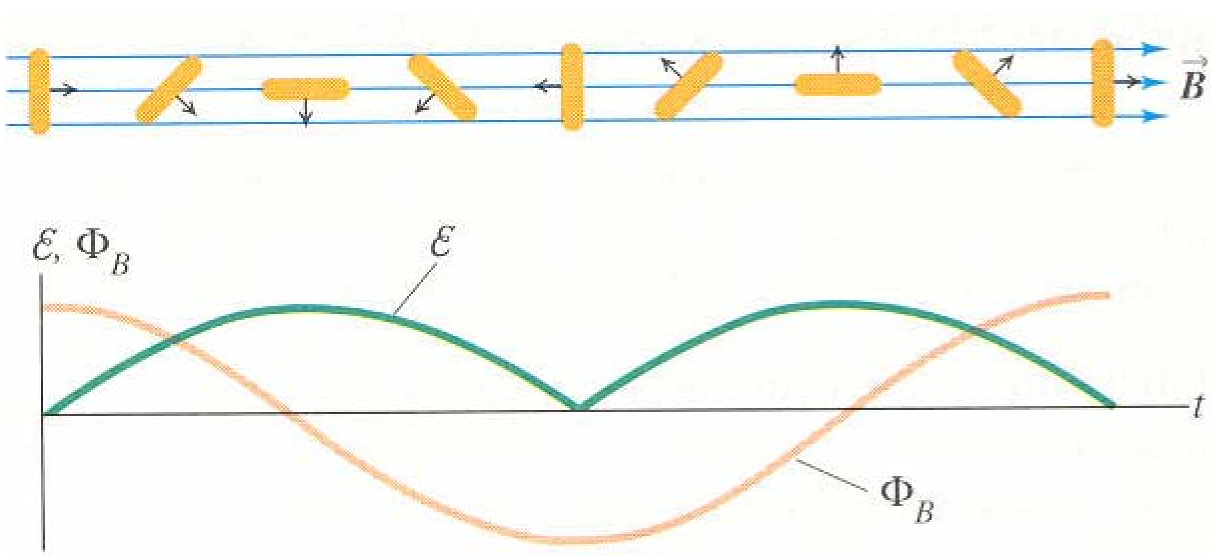


The magnetic field $\vec{B}$ *,and the angular frequency ω, are constant*

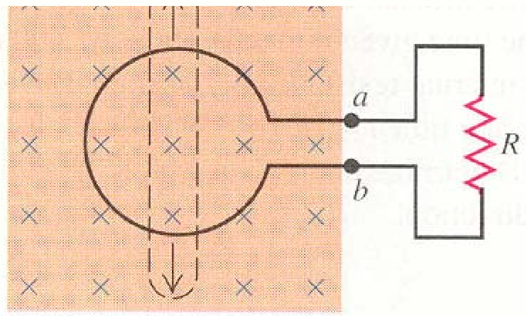
$$\Phi_B = BA\cos\phi = BA\cos\omega t$$

The split-ring commutator "rectifies" the EMF:

$$|\varepsilon| = |\omega AB\cos\omega t| = \omega AB|\cos\omega t|$$

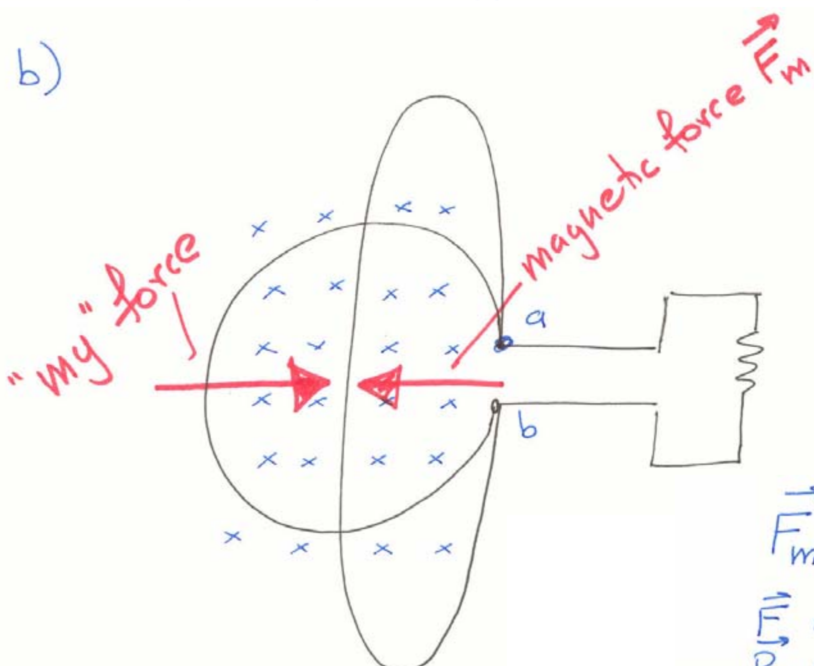


30–35 A flexible circular loop 6.50 cm in diameter lies in a magnetic field with magnitude 0.950 T, directed into the plane of the page in Fig. 30–29. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.250 s. a) Find the average induced emf in the circuit. b) What is the direction of the current in R, from a to b or from b to a? Explain your reasoning.



a) $\bar{\mathcal{E}} = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{\pi r^2 B}{\Delta t} = \dfrac{(3.14)(3.25cm)^2(0.95T)}{(0.25s)} = \boxed{128V}$

b)



$\vec{F}_m = I\vec{\ell} \times \vec{B}$

$\vec{F}$ is to the left

$\vec{B}$ is into the paper

⇓

$\vec{\ell}$ is up in left side of loop

$\boxed{\text{Current in resistor is from a to b}}$

Direction of the Induced EMF's and Currents

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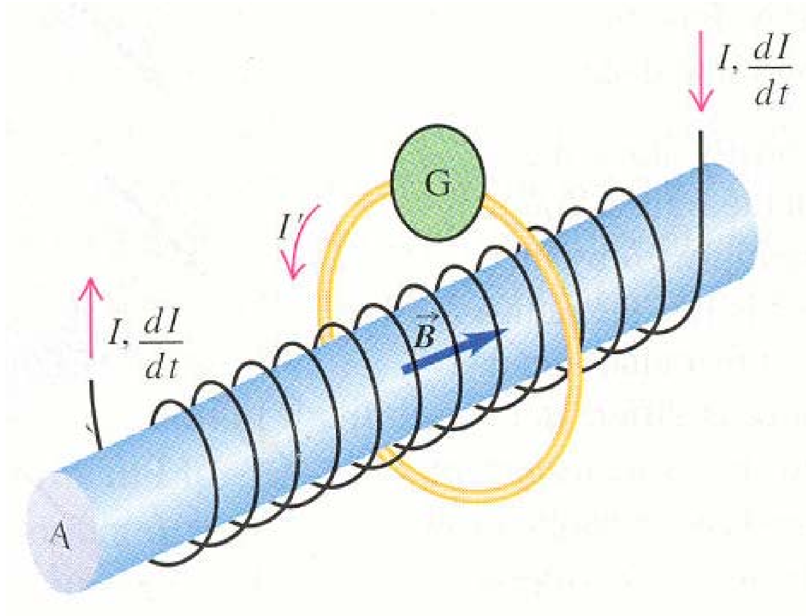
Induced Electric Fields



$$\Phi_B = BA = \mu_0 nIA$$

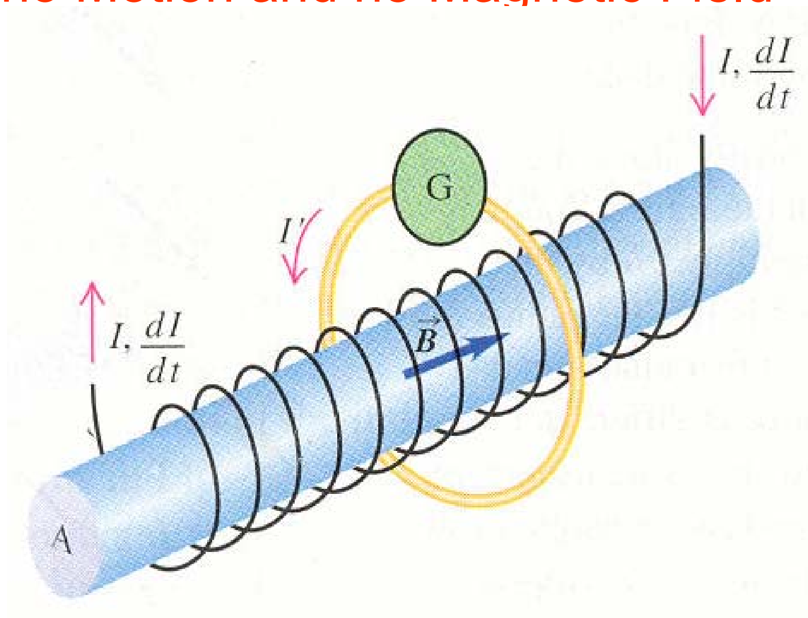
$$\varepsilon = -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt}$$

The EMF will induce a current that will be indicated by the Galvanometer.

This seems very mysterious because there is NO magnetic field outside the solenoid and therefore there can be NO force on the charges inside the conducting loop!

Induced Electric Fields

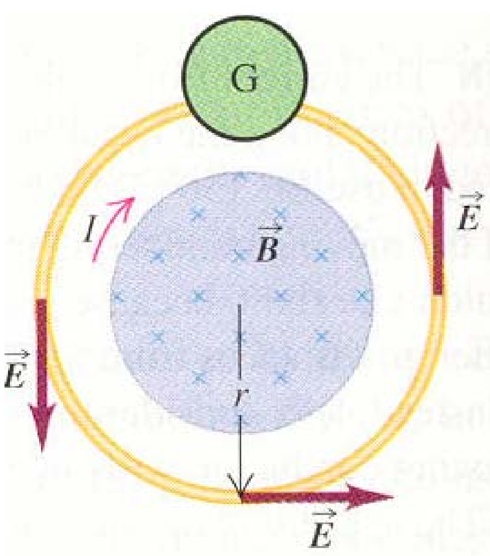
Faraday's Law hold even if there is no Motion and no Magnetic Field



$$\Phi_B = BA = \mu_0 nIA$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt}$$

Faraday's Law implies that there is an "Induced" Electric Field



$$\oint \vec{E}\cdot d\vec{l} = \varepsilon$$

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

This "Induced" Electric Field is a non-electrostatic field that arises, not from static charges, but from a changing B field alone.

Maxwell's Equations

The equations below summarize **all** of the underlying physics of Electricity and Magnetism

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Gauss's Law

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for magnetism

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Ampere's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

AND

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Lorentz Force Law

Maxwell's Equations in Free Space

If there are no charges and no current's, Maxwell's Equations have a very simple and very symmetric form:

$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Note that a changing B will induce an E and a changing E will induce a B. This B can in turn induce an E, which will induce a B, and so on…
It can be shown that these equations predict the existence of a self-sustaining "wave" that propagates with a velocity of:

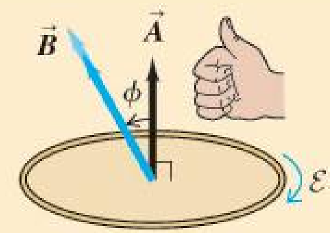
$$v = 1 \Big/ \sqrt{\mu_0 \varepsilon_0}$$

Experimentally this velocity is found to be exactly equal to the speed of light….

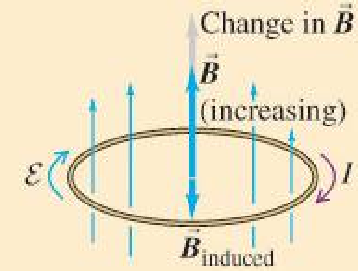
All visible light, as well as radio wave, microwaves, x-rays, gamma rays, ultraviolet and infrared radiation are all electromagnetic in origin!

Faraday's law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop. This relation is valid whether the flux change is caused by a changing magnetic field, motion of the loop, or both. (See Examples 29.1 through 29.7)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{29.3}$$



Lenz's law states that an induced current or emf always tends to oppose or cancel out the change that caused it. Lenz's law can be derived from Faraday's law, and is often easier to use. (See Examples 29.8 and 29.9)



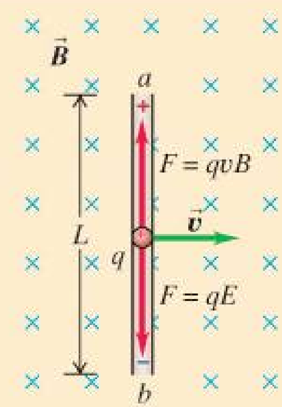
If a conductor moves in a magnetic field, a motional emf is induced. (See Examples 29.10 and 29.11)

$$\mathcal{E} = vBL \tag{29.6}$$

(conductor with length L moves in uniform $\vec{B}$ field, $\vec{L}$ and $\vec{v}$ both perpendicular to $\vec{B}$ and to each other)

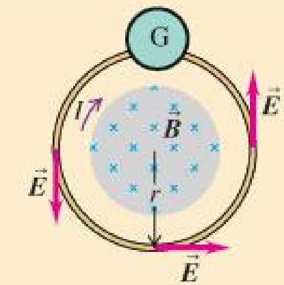
$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \tag{29.7}$$

(all or part of a closed loop moves in a $\vec{B}$ field)



When an emf is induced by a changing magnetic flux through a stationary conductor, there is an induced electric field $\vec{E}$ of nonelectrostatic origin. This field is nonconservative and cannot be associated with a potential. (See Example 29.12)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.10)$$



The relationships between electric and magnetic fields and their sources can be stated compactly in four equations, called Maxwell's equations. Together they form a complete basis for the relation of $\vec{E}$ and $\vec{B}$ fields to their sources.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad (29.18)$$

(Gauss's law for $\vec{E}$ fields)

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad (29.19)$$

(Gauss's law for $\vec{B}$ fields)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad (29.20)$$

(Ampere's law including displacement current)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.21)$$

(Faraday's law)

We have analeyed circuits with

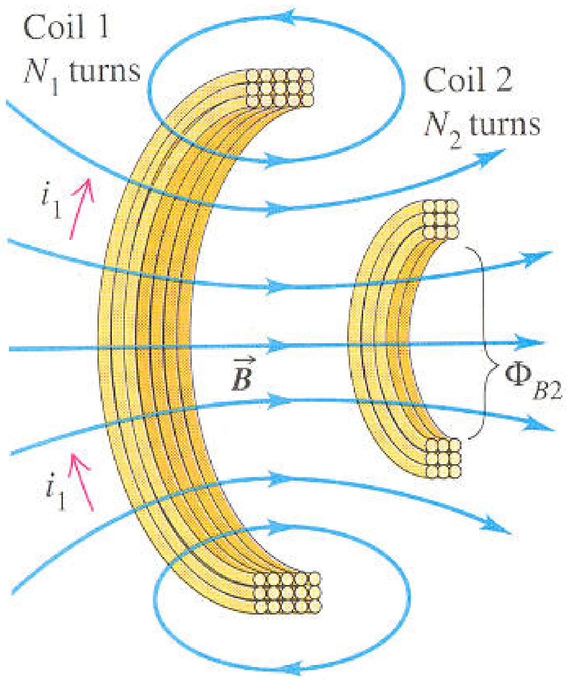
R's

C's

EMF's

We need 1 more circuit element to complete the menagerie

Inductance



If I put a current i_1 in coil 1, the will be some ~~total~~ flux Φ_{B2} in coil 2

If i_1 changes, than Φ_{B2} changes and there is a EMF (remember Mr. Ampere) in loop 2

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$

Φ_{B2} is proportional to i_1 (depends on geometry)

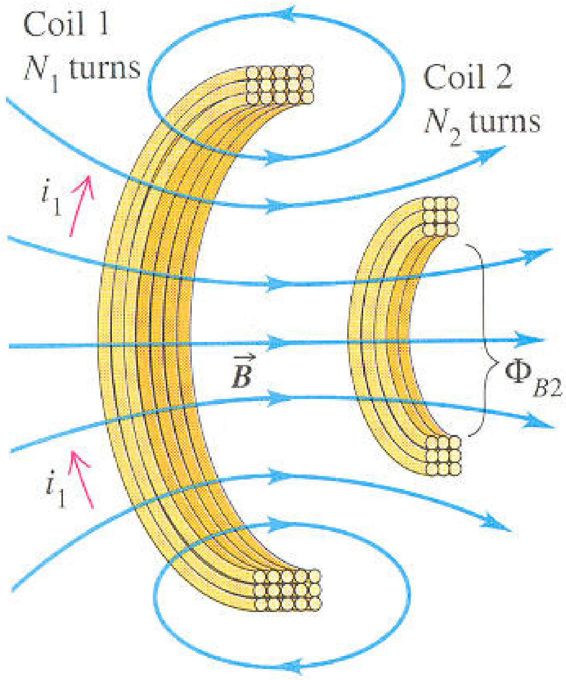
$$N_2 \Phi_{B2} = M_{21} i_1$$

$$N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$$

Mutual Inductance

$$\mathcal{E}_2 = -\boxed{M_{21}} \frac{di_1}{dt}$$

Inductance



$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$$

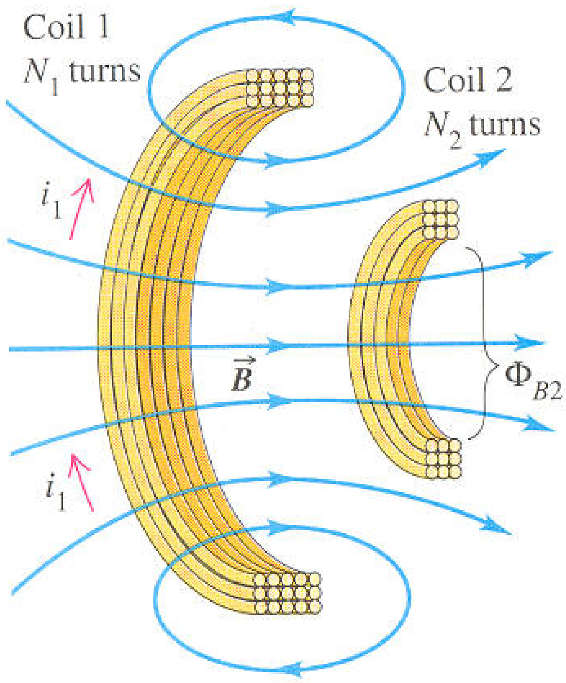
Definition of Mutual Inductance:

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_i}$$

MUTUAL INDUCTANCE DEPENDS ONLY ON THE GEOMETRY OF THE TWO COILS:

- Size
- Shape
- Orientation
- Position

Inductance



Question - Suppose we know M_{21}, what about M_{12}?

It can be shown that for ALL geometries -

$$M_{12} = M_{21} \equiv M$$

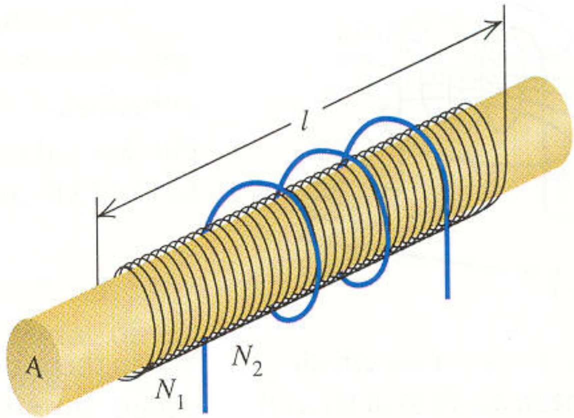
and

$$\mathcal{E}_1 = -M\frac{di_2}{dt} \quad \text{and} \quad \mathcal{E}_2 = -M\frac{di_1}{dt}$$

UNITS - The units of inductance are "Henrys" (H)

$$1\,H = 1(V\cdot s/A) = 1(\Omega\cdot s) = 1(J/A^2)$$

How Big is a "Henry"



$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l} \qquad \Phi = B_1 A$$

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 B_1 A}{i_1} = \frac{N_2}{i_1} \frac{\mu_0 N_1 i_1}{l} A = \frac{\mu_0 A N_1 N_2}{l}$$

In example 31-1 in F & Y

$l = 0.5$, $A = 10\,cm^2$, $N_1 = 1000$ turns, $N_2 = 10$ turns

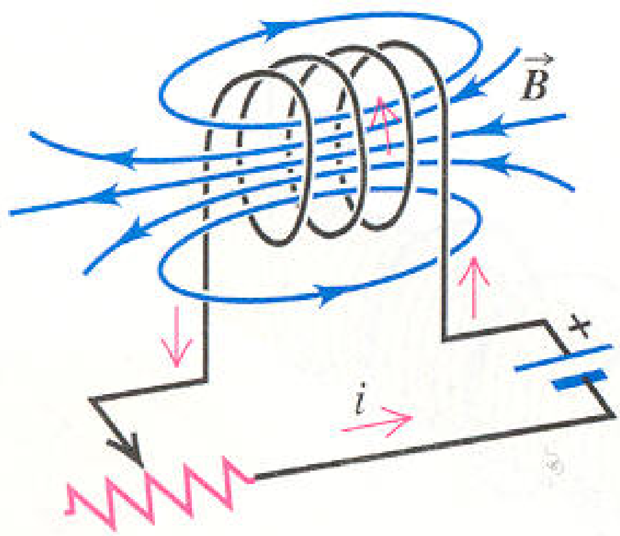
$$M = 25 \times 10^{-6}\ H$$

1 Henry is a HUGE inductance – Most circuit sized inductors are μH

SELF-INDUCTANCE

If I change the current in an isolated coil – I will generate a "SELF INDUCED" emf

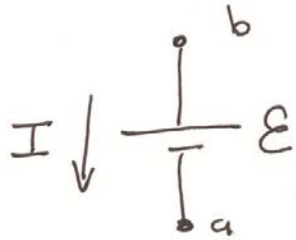
THIS IS A DIRECT RESULT OF LENZ'S LAW



$$L = \frac{N \Phi_B}{i}$$

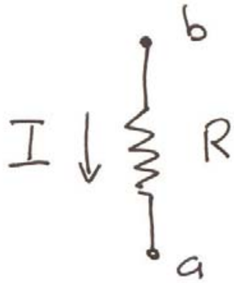
$$\mathcal{E} = -L \frac{di}{dt}$$

Circuit Elements



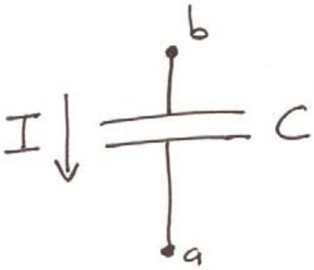
EMF

$$V_{ab} = \mathcal{E}$$



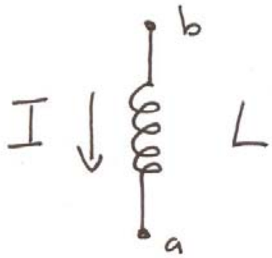
Resistor

$$V_{ab} = IR$$



Capacitor

$$V_{ab} = \frac{1}{C} Q = \frac{1}{C} \int I \, dt$$



Inductor

$$V_{ab} = -L \frac{dI}{dt}$$