Degree of coherence for electromagnetic fields

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Abstract: The relationship between the visibility of fringes and the degree of spatial coherence in electromagnetic two-pinhole interference is assessed. It is demonstrated that the customary definition of the degree of coherence of an electromagnetic field is flawed and a new quantity, free of the formal drawbacks, is introduced. The new definition, which is shown to be consistent with known results for Gaussian statistics, has some unusual properties characteristic only for electromagnetic fields. The degree of coherence is measurable by a sequence of interference experiments.

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1. Introduction

Although Young's double-slit interference experiment has played a pivotal role in the development of optics and quantum physics, the analyses so far have been performed almost exclusively in scalar description. Yet electromagnetic interference and coherence phenomena differ in many fundamental and unexpected ways from the familiar scalar counterparts. Moreover, the electromagnetic theory of optical coherence has become increasingly important not just for the evaluation of polarization properties but due to the recent advances in microstructured materials in general [1–6]. On the other hand, although the coherence studies of electromagnetic fields often deal with paraxial fields or wide-angle far fields, the emergence of nano photonics has given an impetus to comprehensive investigation of general three-dimensional, nonparaxial electromagnetic fields. In particular, it has recently been demonstrated that optical near fields may exhibit remarkable coherence phenomena, which are especially pronounced when resonant surface waves, e.g., surface plasmons or phonons, are excited [7–10].

Unlike in the scalar coherence theory, there does not exist a single scalar quantity that is capable of describing the coherence of electromagnetic fields at two separate space–time points. Hence the correlation properties are thus far examined by using the concept of the degree of polarization, which is capable of describing the correlations at a one point only. In this article, we introduce a scalar quantity describing the second-order correlation properties of electromagnetic fields. We show that this quantity is closely connected to the existing definitions for the degree of polarization and that it has the properties required for the degree of coherence. We also discuss possibilities for its measurement by using simple interference experiments.

2. Young's interference experiment and measures of visibility

We begin by briefly recalling the main aspects of Young's interference experiment both in scalar and electromagnetic descriptions (see Fig. 1). As is usual, the distance d between the aperture plane A and the screen B is assumed to be large compared to the wavelength of the quasi-monochromatic light emitted by the extended source S. In addition, the pinholes P_1 and P_2 are taken to be much smaller than the coherence area of the incoming light and the separation a of the pinholes is much smaller than the distance between the source and the aperture plane A.

If the polarization properties of the field are neglected, the usual scalar approach ensues. It is readily shown that the time-averaged spatial intensity distribution at the screen plane is given by the expression [11]

$$\langle I(\mathbf{r},t)\rangle = \langle I_1(\mathbf{r},t)\rangle + \langle I_2(\mathbf{r},t)\rangle + 2\sqrt{\langle I_1(\mathbf{r},t)\rangle}\sqrt{\langle I_2(\mathbf{r},t)\rangle}\,\Re\left\{\gamma(\mathbf{r}_1,\mathbf{r}_2,(R_1-R_2)/c)\right\}, \qquad (1)$$

where $\langle I_1(\mathbf{r},t)\rangle$ and $\langle I_2(\mathbf{r},t)\rangle$ are the intensities that are measured if only pinhole P_1 or P_2 is open, respectively, R_1 and R_2 are the distances from the pinholes to the observation point, c is

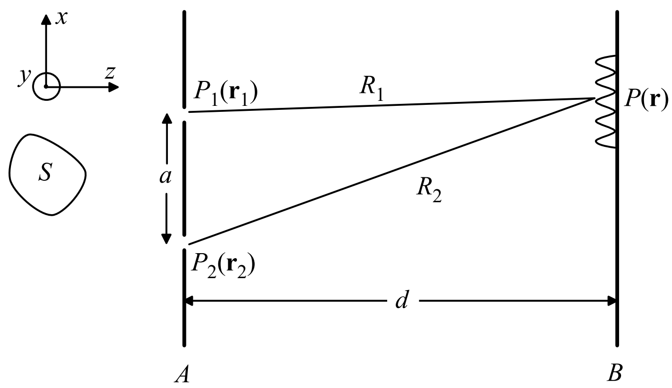

Fig. 1. The geometry of Young's interference experiment.

the speed of light in vacuum, and $\Re$ denotes the real part. The quantity γ appearing in Eq. (1) is called the complex degree of coherence and it is related to the main quantity in scalar coherence theory, namely the mutual coherence function Γ, by the equation

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)}{\sqrt{\langle I(\mathbf{r}_1, t)\rangle \langle I(\mathbf{r}_2, t)\rangle}}, \tag{2}$$

where $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle U^*(\mathbf{r}_1, t)U(\mathbf{r}_2, t+\tau)\rangle$ and $\langle I(\mathbf{r}, t)\rangle = \Gamma(\mathbf{r}, \mathbf{r}, 0)$ is the time-averaged intensity at the position $\mathbf{r}$. Here $U(\mathbf{r}, t)$ denotes a realization of the stationary scalar field. If the intensities at P_1 and P_2 are the same, the absolute value of the complex degree of coherence is equal to the visibility of the interference fringes on the screen. On the other hand, the argument of γ is directly related to the lateral locations of the intensity maxima [11].

The scalar analysis is valid as long as the polarization state of the field is uniform. If, however, the polarization properties vary spatially, the correlations between the electromagnetic field components must be taken into account. In a general situation of arbitrary polarization there are six components of the electromagnetic field, and their correlations can be handled by means of four 3×3 mutual coherence matrices [11,12]. For example, the correlations of the electric field $\mathbf{E}(\mathbf{r}, t)$ are described by the matrix

$$\mathscr{E}(\mathbf{r}_1, \mathbf{r}_2, \tau) = [\mathscr{E}_{ij}(\mathbf{r}_1, \mathbf{r}_2, \tau)] = \left[\left\langle E_i^*(\mathbf{r}_1, t)E_j(\mathbf{r}_2, t+\tau)\right\rangle\right], \tag{3}$$

where the functions $E_i(\mathbf{r}, t)$, $(i = x, y, z)$, denote the Cartesian components of the electric field vector.

When dealing with paraxial electromagnetic fields, as in Fig. 1, the electric mutual coherence matrix reduces to a 2×2 matrix whose elements describe the correlations of, e.g., the x- and y-components only [1]. The interference and coherence properties of paraxial electromagnetic fields may be examined with Young's experiment. Such a situation is thoroughly studied by Karczewski [13], who concluded that Eq. (1) holds for electromagnetic fields as well, assuming that the observation point P is located in the paraxial region. In that case the straightforward electromagnetic extension of the complex degree of coherence of scalar fields is defined by the equation

$$\zeta(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{\operatorname{tr}\mathscr{E}(\mathbf{r}_1, \mathbf{r}_2, \tau)}{\sqrt{\langle I(\mathbf{r}_1, t)\rangle \langle I(\mathbf{r}_2, t)\rangle}}, \tag{4}$$

where tr stands for the trace operation and the time-averaged optical intensity is now given by $\langle I(\mathbf{r}, t)\rangle = \operatorname{tr}\mathscr{E}(\mathbf{r}, \mathbf{r}, 0)$. The quantity ζ is related to both the visibility and the location of the maxima of the interference fringes as in the scalar case.

However, due to the vectorial nature of light, the interference fringes in Young's experiment are not always directly related to the coherence properties of the field. In order to bring out this fact more explicitly, we consider a fully coherent electric field with $\mathbf{E}(\mathbf{r}_1,t) = C\exp(-\mathrm{i}\omega t)\hat{\mathbf{x}}$ and $\mathbf{E}(\mathbf{r}_2,t) = C\exp(-\mathrm{i}\omega t)\hat{\mathbf{y}}$, where C is the complex amplitude of the field, ω denotes the angular frequency, and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors in the x- and the y-directions, respectively. Now the diagonal elements of the electric coherence matrix vanish and thus $\zeta(\mathbf{r}_1,\mathbf{r}_2,\tau) = 0$. This clearly means that no interference fringes are observed. On the other hand, if we examine the same example in a rotated Cartesian coordinate system defined by the unit vectors $\hat{\mathbf{x}}' = 2^{-1/2}(\hat{\mathbf{x}}+\hat{\mathbf{y}})$ and $\hat{\mathbf{y}}' = 2^{-1/2}(-\hat{\mathbf{x}}+\hat{\mathbf{y}})$, we notice that the scalar degree of coherence for the x'- and y'-components of the field take the forms $\zeta_{x'}(\mathbf{r}_1,\mathbf{r}_2,\tau) = \exp(-\mathrm{i}\omega\tau)$ and $\zeta_{y'}(\mathbf{r}_1,\mathbf{r}_2,\tau) = -\exp(-\mathrm{i}\omega\tau)$, respectively. This means, in view of Eq. (1), that the contribution to the interference pattern at the screen from, say the x'-component, is sinusoidal. The same holds also for the y'-component, but the fringes are mutually shifted by half a period so that the resulting intensity distribution is uniform.

Since the field in our example is fully coherent, we must conclude that the quantity ζ (or its space-frequency analog discussed in Refs. [15–17]) does not correctly describe the spatial coherence properties of the field and thus it cannot be called the degree of coherence for the electromagnetic field. However, ζ still has the clear physical meaning that it is directly connected to the visibility of electromagnetic interference fringes.

The fact that the trace of the coherence matrix does not contain information about correlations between the components suggests that the numerator in Eq. (4) is not invariant under transformations into orthogonal curvilinear coordinate systems [18], such as circular cylindrical or spherical polar coordinates. This prediction may be understood when we recall that such a transformation can be expressed in the form of a position-dependent rotation matrix $\mathscr{T}(\mathbf{r})$, i.e., any (column) vector in the new basis takes the form $\mathbf{F}'(\mathbf{r}) = \mathscr{T}(\mathbf{r})\mathbf{F}(\mathbf{r})$. Since the transformation matrix is orthogonal, the mutual coherence matrix changes into

$$\mathscr{E}'(\mathbf{r}_1,\mathbf{r}_2,\tau) = \mathscr{T}(\mathbf{r}_1)\mathscr{E}(\mathbf{r}_1,\mathbf{r}_2,\tau)\mathscr{T}^{-1}(\mathbf{r}_2). \tag{5}$$

We see at once that ζ of Eq. (4) is not invariant under the transformation, except in the special case of $\mathscr{T}(\mathbf{r}_1) = \mathscr{T}(\mathbf{r}_2)$, which occurs, for example, in a pure rotation of the coordinate system. Although in many cases the most natural coordinate system to be used is Cartesian, there exist several situations in which, for example, the spherical polar coordinates are the best choice for describing the behavior of the field. Such a situation is encountered when examining far-field radiation patterns of electromagnetic sources [15, 16].

3. Electromagnetic degree of coherence

Our physical examples and mathematical arguments clearly show that if we want to characterize the coherence of the electromagnetic field by a single scalar quantity [13], the visibility of interference fringes in a single experiment cannot generally be used for this purpose. Hence the definition of such a quantity should be approached from a different point of view. In addition, since a general electromagnetic field is not paraxial we must employ the full 3×3 electric coherence matrix.

Let us introduce a quantity $\gamma_{\mathscr{E}}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ by the equation

$$\begin{aligned}\gamma_{\mathscr{E}}^2(\mathbf{r}_1,\mathbf{r}_2,\tau) &= \frac{\mathrm{tr}\,[\mathscr{E}(\mathbf{r}_1,\mathbf{r}_2,\tau)\mathscr{E}(\mathbf{r}_2,\mathbf{r}_1,-\tau)]}{\langle I(\mathbf{r}_1,t)\rangle\langle I(\mathbf{r}_2,t)\rangle} \\ &= \frac{\sum_{i,j}|\mathscr{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)|^2}{\sum_{i,j}\mathscr{E}_{ii}(\mathbf{r}_1,\mathbf{r}_1,0)\mathscr{E}_{jj}(\mathbf{r}_2,\mathbf{r}_2,0)},\end{aligned} \tag{6}$$

where $i, j = x,y,z$, and we have made use of the Hermiticity relation $\mathscr{E}^*_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau) = \mathscr{E}_{ji}(\mathbf{r}_2,\mathbf{r}_1,-\tau)$ satisfied by the electric coherence-matrix elements. Thus, the quantity $\gamma_{\mathscr{E}}$, which we shall refer to as the degree of coherence for electromagnetic fields, is equal to the Frobenius (or Euclidean) norm [19] of the electric coherence matrix $\mathscr{E}$, normalized by the factor $\langle I(\mathbf{r}_1,t)\rangle^{1/2}\langle I(\mathbf{r}_2,t)\rangle^{1/2}$.

Since the elements of the electric mutual coherence matrix satisfy the inequality $|\mathscr{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)|^2 \leq \mathscr{E}_{ii}(\mathbf{r}_1,\mathbf{r}_1,0)\mathscr{E}_{jj}(\mathbf{r}_2,\mathbf{r}_2,0)$ derived from the nonnegative definiteness condition [11, 20], $\gamma_{\mathscr{E}}$ is immediately seen to obey the condition $0 \leq \gamma_{\mathscr{E}} \leq 1$. Unlike the quantity ζ defined in Eq. (4), $\gamma_{\mathscr{E}}$ contains information about the correlations between the Cartesian components of the field and it equals unity if, and only if, there is a perfect correlation between all the field components at $\mathbf{r}_1$ and $\mathbf{r}_2$. Thus $\gamma_{\mathscr{E}}$ is always equal to one for fully coherent fields and hence also for the example considered in Section 2, for which ζ was found to be zero.

It is also noticed that, whereas γ is generally a complex quantity, $\gamma_{\mathscr{E}}$ is real. This property has its roots in the fact that the arguments (or phases) of the electric coherence-matrix elements are, in general, mutually independent. Hence it is not possible to define a single complex number which retains the phase information of all elements of the matrix. It should be kept in mind, however, that in scalar coherence theory it is the absolute value, rather than the phase, of the complex degree of coherence which gives the measure for strength of the field correlations. Therefore, the newly defined quantity $\gamma_{\mathscr{E}}$ may be seen as an extension of the absolute value of the complex degree of coherence for scalar fields. This can also be verified immediately by retaining only one field component in Eq. (6).

Let us next consider the transformation of $\gamma_{\mathscr{E}}$ into an orthogonal curvilinear coordinate system. By inserting Eq. (5) into Eq. (6), we immediately observe that, unlike ζ of Eq. (4), the electromagnetic degree of coherence $\gamma_{\mathscr{E}}$ is invariant under such a transformation. This means that the degree of coherence may be calculated by using, for example, spherical polar coordinates, which is useful when examining far-field radiation patterns of partially coherent sources. On the other hand, this result implies that the rotation of the field at either $\mathbf{r}_1$ or $\mathbf{r}_2$ by using a suitable optical element will not affect the value of $\gamma_{\mathscr{E}}$, which is of importance to remember when one performs two-pinhole interference experiments, for instance.

Since the degree of coherence contains information about the correlations that exist between the orthogonal components of the electric field at a pair of points, one might expect that there exists a connection between $\gamma_{\mathscr{E}}$ and the degree of polarization that characterizes correlations in a single point. By setting $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$ and $\tau = 0$, the electric coherence matrix of Eq. (3) reduces to the 3×3 equal-time coherence matrix, which we denote by $\Phi(\mathbf{r})$. Thus, the space–time counterpart of the degree of polarization, $P_3(\mathbf{r})$, for three-dimensional fields [10, 21–23] takes on the form

$$P_3^2(\mathbf{r}) = \frac{3}{2}\left[\frac{\mathrm{tr}\Phi^2(\mathbf{r})}{\mathrm{tr}^2\Phi(\mathbf{r})} - \frac{1}{3}\right] = \frac{3}{2}\left[\gamma^2_{\mathscr{E}}(\mathbf{r}) - \frac{1}{3}\right], \tag{7}$$

where $\gamma_{\mathscr{E}}(\mathbf{r}) = \gamma_{\mathscr{E}}(\mathbf{r},\mathbf{r},0)$. Furthermore, for paraxial fields, connection to the conventional two-dimensional degree of polarization [11], $P_2(\mathbf{r})$, is established

$$P_2^2(\mathbf{r}) = 2\left[\frac{\mathrm{tr}\Phi^2(\mathbf{r})}{\mathrm{tr}^2\Phi(\mathbf{r})} - \frac{1}{2}\right] = 2\left[\gamma^2_{\mathscr{E}}(\mathbf{r}) - \frac{1}{2}\right]. \tag{8}$$

Now an important property of the degree of coherence of electromagnetic fields emerges. Namely, unlike in the scalar case, Eq. (6) does not approach unity when the two points coincide. At first sight this might seem quite counter-intuitive, but it is, in fact, as expected since the numerator of Eq. (6) contains cross-correlation functions characterizing coherence between orthogonal field components. Thus, the value of $\gamma_{\mathscr{E}}(\mathbf{r})$ is determined by the polarization state of the field, as evidenced by Eqs. (7) and (8), and it is equal to one only for fully polarized fields.

This is true both in two or three dimensional description of partial polarization. Moreover, due to the autocorrelation functions the value of $\gamma_{\mathscr{E}}(\mathbf{r})$ never assumes the value of zero. In fact, we see from Eq. (7) that the minimum value for $\gamma_{\mathscr{E}}(\mathbf{r})$ is $1/\sqrt{3}$.

4. Measurements of the degree of coherence

Let us recall from Ref. [11, Chap. 8] that, for fields obeying Gaussian statistics, there exists a simple connection between the intensity fluctuations $\Delta I(\mathbf{r},t) = I(\mathbf{r},t) - \langle I(\mathbf{r},t)\rangle$ and the scalar degree of coherence. More specifically, the square of the absolute value of γ may be expressed in the form

$$|\gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)|^2 = \frac{\langle \Delta I(\mathbf{r}_1,t)\Delta I(\mathbf{r}_2,t+\tau)\rangle}{\langle I(\mathbf{r}_1,t)\rangle\langle I(\mathbf{r}_2,t)\rangle}. \tag{9}$$

Since the right-hand side contains only terms depending on the intensities at $\mathbf{r}_1$ and $\mathbf{r}_2$ which have the same physical meaning in both scalar and electromagnetic cases, it is reasonable to demand that the functional form of Eq. (9) must be the same for electromagnetic fields. Hence, under the assumption of Gaussian statistics, Eq. (9) may be understood to be the definition of the degree of coherence, not only with the scalar approach, but in the electromagnetic case as well.

Let us now make use of the result for electromagnetic fields obeying Gaussian statistics, namely (see Ref. [11], Eq. (8.4–15))

$$\langle \Delta I(\mathbf{r}_1,t)\Delta I(\mathbf{r}_2,t+\tau)\rangle = \sum_{i,j} \left|\mathscr{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right|^2. \tag{10}$$

Substitution of this into Eq. (9) immediately yields that the expression of the degree of coherence obtained in electromagnetic Gaussian statistics coincides with our general formula given in Eq. (6). Therefore our definition of $\gamma_{\mathscr{E}}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is the only possible one that is fully consistent with the scalar degree of coherence (or its absolute value), of course assuming that the coherence properties of only the electric field are concerned.

The straightforward connection between the degree of coherence and the intensity fluctuations established above predicts the possibility for direct measurement of the degree of coherence, equivalently to the scalar case [11]. This applies naturally only to fields obeying Gaussian statistics and if the statistical properties of the field are not Gaussian, other ways for measuring $\gamma_{\mathscr{E}}$ must be found. Since we already know that with Young's two-pinhole experiment a single measurement of the visibility of the interference fringes does not correctly predict the electromagnetic degree of coherence, we suggest the following series of four measurements, in which again attention is restricted to two-dimensional fields only.

In the first step, a linear polarizer is used to filter one component, say the y-component, of the field. The value of $\mathscr{E}_{xx}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is then obtained, similarly to the scalar case, by a direct measurement of the visibility of the interference fringes. In the second step, the polarizer is rotated by $\pi/2$ radians in order to block the x-component and the element $\mathscr{E}_{yy}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is obtained analogously to the step one. In steps three and four, which are essentially similar to that discussed recently by Gori [1], the x- and y-components of the field at the pinhole P_2 are at first rotated by $\pi/2$ radians by an appropriate optical component. In the third step, after the rotation, a linear polarizer is used to filter out the field's y-component, in which case the fringe visibility gives the element $\mathscr{E}_{xy}(\mathbf{r}_1,\mathbf{r}_2,\tau)$. In the fourth step, the x-component of the field is cut off by the polarizer and the value of $\mathscr{E}_{yx}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is obtained. The degree of coherence $\gamma_{\mathscr{E}}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is then calculated by using Eq. (6).

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5. Conclusions

We have analyzed the coherence properties of nonuniformly polarized electromagnetic fields. We showed that the customary definition for the degree of coherence for the electromagnetic fields does not predict the coherence properties accurately and may, in fact, lead to severe misinterpretations of spatial coherence. This phenomenon arises from the fact that a quantity derived straightforwardly from the degree of coherence for scalar fields is connected to the visibility of the interference fringes similarly to the scalar case. In the electromagnetic case the visibility is affected, not only by the coherence properties of the field, but by the polarization properties of the field as well. Therefore the correlations between the electric-field components can not be neglected in the analysis of nonuniformly polarized partially coherent fields.

In our new definition for the degree of coherence, all the elements of the electric coherence matrix describing the correlations between the Cartesian components of the field are taken into account. Hence the degree of coherence remains invariant if the field is, for example, rotated by using a suitable anisotropic element. On the other hand, our quantity remains invariant in the rotation of the coordinate axes, as well as in various transformations between the coordinate systems which should be particularly useful when examining the far-field coherence of electromagnetic sources.

In this article we have considered the correlations of the electric field only. However, one may equivalently define a quantity describing the correlations of the magnetic field. On the other hand, Eq. (6) may readily be extended to take into account also the correlations that exist between the electric and magnetic field components.

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