Typeset in November 2002

Chapter III Classical Electrodynamics

What is light? The study of relativity left us completely in the dark, even though we had embarked in it for precisely that aim. True, we have learned how the motion of light compares to that of objects. We also learned that light is that moving entity which cannot be stopped; but we haven't learned anything about its own nature. The answer to this old question emerges only from the study of those types of motion which are *not* related to gravitation, such as the way magicians levitate objects.

13. Liquid electricity, and invisible fields

Revisiting the list of of motors one finds in this world, one remarks that gravitation does not describe almost any of them. Neither the motion of see waves, of fire, of earthquakes, nor that of a gentle breeze are due to gravity. The same applies to the motion of muscles. Have you ever listened to your own heart beat, e.g. with a stethoscope?Without having done so, nobody can pretend to have experienced the mystery of motion. You have about 3000 million beats in your lifetime. Then they stop.

See page 106

Challenge 708

It was one of the most astonishing discoveries of science that all these and most other cases of everyday motion, as well as the nature of light itself, are connected to observations performed already thousands of years ago with two strange stones. These stones show that all examples of motion which are called *mechanical* in everyday life, are, without exception, of *electrical* origin.

In particular, the solidity of matter, its softness and its impenetrability are due to internal electricity; also the emission of light is. As these aspects are part of everyday life, we leave aside all complications due to gravity and curved space-time. Again, the most productive way to proceed is to study first, like in the case of gravity, those types of motion which are generated without any contact between the involved bodies.

Amber, lodestone, and mobile phones

Any fool can ask more questions
than seven sages can answer.

The story of electricity starts with trees. Trees have a special relation to electricity. When one cuts a tree, a viscous resin appears. With time it solidifies, and after millions of years it forms *amber*. When one rubs amber with a cat fur, it acquires the ability to attract small objects, such as saw dust or pieces of paper. This was already known to Thales of Miletus, one of the original seven sages, in the sixth century BCE. The same observation can be made with many other polymer combinations, for example with combs and hair, with shoe soles and carpets, or with TV screens and dust. Even children are always surprised by the effect a rubbed comb has on a running water tap. Challenge 709

The other part of the story is about an iron mineral found in certain caves around the world, e.g. in Greece, in the province of Thessalia, in a region (still) called Magnesia, or in China. When one puts two stones of this mineral near each other, they attract or repel each other, depending on their relative orientation. In addition, these stones attract objects made of cobalt, nickel, or iron.

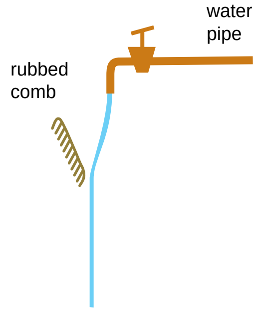

Figure 145 How to amaze kids

Today one also finds various little objects in nature with more sophisticated properties. Some are able to switch on televisions, others unlock car doors, still others allow to talk with far away friends.

All these observations show that in nature there are situations where bodies exert influence on others *at a distance*. The space surrounding a body with such an influence is said to contain a field. A *(physical) field* is thus an entity which manifests itself by accelerating other bodies in that region of space. A field is some 'stuff' taking up space, but obviously having no mass. The field surrounding the mineral found in Magnesia is called a *magnetic field* and the stones themselves *magnets*.* The field around amber – called ελεκτρον in Greek, from a root meaning 'brilliant, shining' – is called an *electric field*. The name is due to a proposal by the famous English part-time physicist William Gilbert (1544–1603) who was the physician of Queen Elizabeth. Objects surrounded by a permanent electric field are called *electrets*. They are much less common than magnets; among others, they are used in certain loudspeaker systems.**

The field around a mobile phone is called a *radio* field, or as we will see later, an *electromagnetic* field. We will find out later that many other objects are surrounded by such fields, though often very weak. Objects such as mobile phones are called radio transmitters or radio emitters.

* A pretty book about the history of magnetism and the excitement it generates is JAMES D. LIVINGSTON, *Driving force – the natural magic of magnets*, Harvard University Press, 1996.
** The Kirlian effect, which allows to make so intriguingly beautiful photographs, is due to a time-varying electric field.

Search	Magnetic charge
Smallest magnetic charge suggested by quantum theory	$g = \frac{h}{e} = \frac{eZ_0}{2\alpha} = 4.1\,\mathrm{pWb}$
Search in minerals	none Ref. 369
Search in meteorites	none Ref. 369
Search in cosmic rays	none Ref. 369
Search with high energy accelerators	none Ref. 369

Table 32 Some searches for magnetic monopoles, i.e., for magnetic charges

Observation	Magnetic field
Lowest measured magnetic field	ca. 1 fT
Magnetic field produced by brain currents	ca. 0.1 pT to 3 pT
Intergalactic magnetic fields	1 pT to 10 pT
Magnetic field in the human chest, due to heart currents	ca. 100 pT
Magnetic field of our galaxy	0.5 nT
Magnetic field of earth	20 μT to 70 μT
Magnetic field below high voltage power line	ca. $10^{-..}$ T
Magnetic field inside modern home	10^{-7} T to 10^{-4} T
Magnetic field near mobile phone	ca. $10^{-..}$ T
Magnetic field in light beam	... T
Magnetic field near iron magnet	100 mT
Solar spots	ca. 1 T
Magnetic fields near high tech permanent magnet	max 1.3 T
Magnetic fields in particle accelerator	ca. 10 T
Maximum magnetic field produced with superconducting coils	22 T
Highest long time static magnetic fields produced in laboratory using hybrid magnets	50 T
Highest *pulsed* magnetic fields produced without coil destruction	74 T
Pulsed magnetic fields produced, during about 1 μs, using imploding coils	ca. 1000 T
Field on neutron star	from 10^6 T to 10^{11} T
Quantum critical magnetic field	ca. $6 \cdot 10^9$ T
Highest field ever measured, on magnetar SGR-1806-20	$0.8 \cdot 10^{11}$ T
Maximum (Planck) magnetic field	$2.2 \cdot 10^{53}$ T

Table 33 Some observed magnetic fields

Fields influence other bodies over a distance, without any material support. For a long time, this was quite rare in everyday life, as laws in most countries have strict upper limits for machines using and producing such fields. For any device which moves, produces sounds, or creates moving pictures, the fields are usually required to remain inside them. For this reason magicians moving an object on a table via a hidden magnet still continue to surprise and entertain their public. To feel the fascination of fields more strongly, a deeper look into a few experimental results is worthwhile.

How can one make lightnings?

Everybody has seen a lightning or has observed the effect it can have when hitting a tree. Obviously lightning is a moving phenomenon. Photographs show that their tips advance with a speed of over 10^5 m/s. But *what* is moving? To find out, one has to find a way to make lightnings oneself.

Figure yet to be included

Figure 146 Lightning: a picture taken with a moving camera, showing the multiple strokes it consists of

In 1995, the car company General Motors accidentally rediscovered an old and simple method for achieving this. They had inadvertently build a spark generating mechanism into their cars; when filling the tank with fuel, sparks were
Ref. 370 generated which sometimes lead to the explosion of the fuel. They had to recall 2 million vehicles of its Opel brand. What had they done?

The engineers had unknowingly copied the conditions for a electrical device which everybody can build at home and which was originally invented by William Thomson.* Repeating his experiment today, one would take a few water taps, four empty bean or coffee cans, of which two have been opened
Ref. 371 at both sides, some nylon rope and some metal wire.

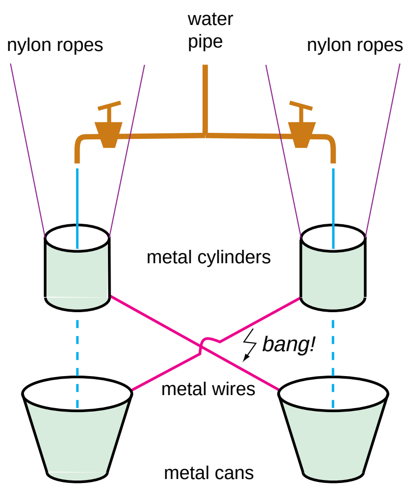

Figure 147 A simple Kelvin generator

Putting all together as shown in Figure 147 and letting the water flow, one finds a strange effect: strong sparks periodically jump between the two copper wires at the point where they are nearest to each other, making loud bangs. Can you guess what condition for the flow has to be realized for this to
Challenge 710 work? And what Opel did to repair the cars?

If one stops the water flow just before the next spark is due, one finds that both buckets attract saw-dust and pieces of paper. The generator thus does

* William Thomson (1824–1907), important unionist Irish physicist, professor in Glasgow. He worked on the determination of the age of the earth, showing that it was much older than 6000 years, as several sects believed; he strongly influenced the development of the theory of magnetism and electricity, the description of the aether, and thermodynamics. He propagated the use of the term 'energy' as it is common today, instead of the unclear older terms. He was one of the last scientists propagating mechanic analogies for the explanation of phenomena, and thus strongly opposed Maxwell's description of electromagnetism. Probably for this reason he did not receive a Nobel prize. He also was one of the minds behind the laying of the first transatlantic telegraphic cable. Victorian to his bones, when he was was made a Lord, he chose the name of a small brook near his home as his new name; thus he became Lord Kelvin of Largs. Therefore the temperature unit got its name from a small English river.

the same that rubbing amber does, just with more bang for the buck(et). Both buckets are surrounded by electric fields. The field increases with time, until the spark jumps. Just after the spark, the buckets are (almost) without electric field. Obviously, the flow of water somehow builds up an entity on each bucket, today called *electric charge*, which can flow in metals and, when the fields are high enough, through air. One also finds that the two buckets are surrounded by two different types of electric fields: bodies which are attracted by one bucket are repelled by the other. All other experiments confirm that there are *two* types of charges. The US politician and part-time physicist Benjamin Franklin (1706–1790) called the electricity created on a glass rod rubbed with a dry cloth *positive*, the one on a piece of amber *negative*. (Before him, the two types of charges used to be called called 'vitreous' and 'resinous'.) Bodies with charges of the same sign repel each other, bodies with opposite charges attract each other; charges of opposite sign flowing together cancel each other out.*

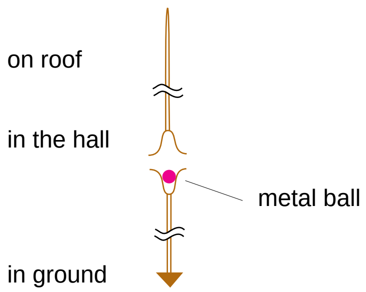

Figure 148 Franklin's personal lightning rod

In summary, electric fields start at bodies, provided they are charged. Charging is possible by rubbing and similar processes. Charge can flow, and then is called electric current. The worst conductors of current are polymers; they are often called insulators. Metals are the best conductors, especially silver and copper. This is the reason that at present, after hundred years of use of electricity, the highest concentration of copper in the world is below the surface of Manhattan.

Of course, one has to check whether real thunderstorm lightnings actually are electrical in origin. In 1752, experiments performed in France, following a suggestion of Benjamin Franklin published in London in 1751, showed that one can indeed draw electricity from thunderstorms via a long rod.** These French experiments rendered Franklin world famous; they also started the use of lightning rods throughout the world. Later on, Franklin had a lightning rod built through his own house, but of a somewhat unusual type, as shown in Figure 148. Can you guess what it did in his hall during bad weather, all parts being made of metal? Challenge 711

What is electric charge?

If all experiments with charge can be explained by calling the two charges positive and negative, the implication is that some bodies have more, and some less charge than the uncharged, *neutral* ones. Electricity thus only flows when two differently charged bodies are brought into contact. Now, if charge can flow and accumulate, one must be able to somehow measure its amount. Obviously, the *amount* of charge on a body, usually abbreviated q, is

* In fact, there are many other ways to produces sparks or even *arcs*, i.e. sustained sparks; there is even a complete subculture of people who do this as a hobby at home. Those who have a larger budget do it professionally, in particular accelerators. See the http://www.mathematik.uni-marburg.de/ kronjaeg/hv/index.html web site.
** There is still research going on into the details of how lightnings are generated and how they propagate. A little about this topic is said on page 397.

This is a section of the freely downloadable e-textbook

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To the kind reader

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- What was hard to understand?
- Did you find any mistakes?
- What figure or explanation were you expecting?
- Which page was boring?

Of course, any other suggestion is welcome. This section is part of a physics text written over many years. The text lives and grows through the feedback from its readers, who help to improve and to complete it. For a particularly useful contribution (send it in english, italian, dutch, german, spanish, portuguese, or french) you will be mentioned in the foreword of the text, or receive a small reward, or both.

Enjoy!

Christoph Schiller
mm@motionmountain.net

Charges	Physical property	Mathematical name (see later for definitions)
can be compared	distinguishability	set
can be ordered	sequence	order
can change gradually	continuity	completeness
can be stored	accumulability	additivity
don't change	conservation	invariance
can be divided	separability	positive or negative

Table 34 Properties of classical electric charge

Observation	Charge
Smallest known non-vanishing charge	$0.5 \cdot 10^{-19}$ C
Charge per bit in computer memory	10^{-13} C
Charge in small capacitor	10^{-7} C
Charge flow in average lightning stroke	1 C to 100 C
Charge stored in a full car battery	0.2 MC
Charge of planet earth	ca. 1 MC
Charge separated by modern power station in one year	ca. $3 \cdot 10^{11}$ C
Total charge of one sign observed in universe	ca. $10^{62\pm2}$ C

Table 35 Values of electrical charge observed in nature

defined via the influence the body, say a piece of saw dust, feels when subjected to a field. Charge is thus defined by comparing it to a standard reference charge. For a charged body of mass m accelerated in a field, its unknown charge q is determined by the relation

$$\frac{q}{q_{\text{ref}}} = \frac{ma}{m_{\text{ref}} a_{\text{ref}}} \quad , \tag{327}$$

i.e., by comparing it to the acceleration and mass of the reference charge. This definition reflects the observation that mass alone is not sufficient for a complete characterization of a body. For a full description of motion one needs to know its electric charge; charge is therefore the second intrinsic property of bodies we discover in our walk.

By the way, the unit of charge, the *coulomb*, is nowadays defined through a standard flow through metal wires, as explained in Appendix B. This is possible because all experiments show that charge is conserved, that it flows, that it flows continuously, and that it can accumulate. Charge thus behaves like a fluid substance. Therefore one is forced to use for its description a scalar quantity q, which can take positive, vanishing, or negative values.

In everyday life these properties of electric charge, listed also in Table 34, describe observations with sufficient accuracy. But as in the case of all previously encountered classical concepts, these experimental results about electrical charge will turn out to be only approximate. More precise experiments will require a revision of several properties.

Experiments show that the entity which accelerates charged bodies, the *electric field*, behaves like a little arrow fixed at each place $\mathbf{x}$ in space; its length and its direction does not depend on the observer. In short, the electric field $\mathbf{E}(\mathbf{x})$ is a *vector* field. Experiments show

Observation	Electric field
Cosmic noise	ca. $10\,\mu\mathrm{V/m}$
Field 1 m away from an electron	...
Field of a 100 W FM radio transmitter at 100 km distance	$0.5\,\mathrm{mV/m}$
Field in solar wind	...
Field in clouds	...
Field inside conductors, such as copper wire	$0.1\,\mathrm{V/m}$
Field of a 100 W bulb at 1 m distance	$50\,\mathrm{V/m}$
Ground field in earth's atmosphere	100 to $300\,\mathrm{V/m}$
Maximum electric field in air before sparks appear	ca. $1\,\mathrm{MV/m} = 1\,\mathrm{kV/mm}$
Electric fields in biological membranes	$10\,\mathrm{MV/m}$
Electric fields inside capacitors	up to $1\,\mathrm{GV/m}$
Electric fields in most intense laser beams	$100\,\mathrm{TV/m}$
Electric fields in U^{91+} ions, at nucleus	$1\,\mathrm{EV/m}$
Maximum electric field in vacuum, limited by pair production	$1.3\,\mathrm{EV/m}$
Planck electric field	$6.5 \cdot 10^{61}\,\mathrm{V/m}$

Table 36 Some observed electric fields

that it is best defined by the relation

$$q\mathbf{E}(\mathbf{x}) = m\mathbf{a}(\mathbf{x}) \tag{328}$$

taken at every point in space **x**. The definition of the electric field is thus indeed based on how it *moves* charges.* The field is measured in multiples of the unit N/C or the identical unit V/m.

Challenge 714

To describe motion due to electricity completely, one also needs a relation explaining how charges *produce* electric fields. This relation was first established with precision by Charles-Augustin de Coulomb in his private estate, during the French revolution.** He found that around a small or spherical charge Q *at rest* there is an electric field given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\frac{\mathbf{r}}{r} \quad \text{where} \quad \frac{1}{4\pi\varepsilon_o} = 8.9\,\mathrm{GVm/C} \quad . \tag{329}$$

Later on we will extend the relation for a charge in motion. The strange proportionality constant is due to the historical way the unit of charge was defined first.*** The essential point of the formula is the decrease of the field with the square of the distance; can you imagine the origin of this dependence?

Challenge 715

Challenge 713

* Does the definition of electric field given here assume a speed of the charge much smaller than that of light?

** Charles-Augustin de Coulomb (1736, Angoulême–1806, Paris), French engineer and physicist. His careful experiments on electric charges provided the basis for the study of electricity.

*** Other definitions of this and other proportionality constants to be encountered later are possible, leading to *unit systems* different from the SI system used here. The SI system is presented in detail in Appendix B. Among the older competitors, the gaussian unit system often used in theoretical calculations, the Heaviside-Lorentz unit system, the electrostatic unit system, and the electromagnetic unit system are the most important ones. For more details, see the standard text by J.D. Jackson, *Classical electrodynamics*, 3rd edition, Wiley, 1998, The two previous equations allow to write the interaction between two charged bodies as

$$\frac{d\mathbf{p}_1}{dt} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\frac{\mathbf{r}}{r} = -\frac{d\mathbf{p}_2}{dt} \tag{330}$$

where $\mathbf{p}$ is the momentum change, and $\mathbf{r}$ is the vector connecting the two centres of mass. This famous expression for electrostatic attraction and repulsion, also due to Coulomb, is valid only for small or for spherical charged bodies *at rest*.

The strength of this interaction is considerable. For example, it is the basis for the force of our muscles. Their force is a macroscopic effect of this equation. Another example is provided by the strength of steel or diamond. As we will discover, its atoms, and those of any other material, are kept together by electrostatic attraction. As a final example to convince yourself of the strength of electrostatic attraction, answer the following: What is the force between two boxes with a gram of protons each, located on the two poles of the earth? Try to guess the result,before you calculate the astonishing value. Challenge 716

Due to the strength of electromagnetic interactions, separating charges is not an easy task. This is the reason that electrical effects are in common use only for about a hundred years. One had to wait for the invention of practical and efficient devices for separating charges and putting them into motion. Of course this implies the use of energy. Batteries, as used e.g. in portable phones, use chemical energy to do the trick,* thermoelectric elements, as used in some watches, use the temperature difference between the wrist and the air to separate charges, solar cells use light, and dynamos or the Kelvin generator use kinetic energy.

Do uncharged bodies attract each other? In first approximation they do not. But when studying the question more precisely, they can attract each other. Can you find the conditions for this to happen?In fact, the conditions are quite important, as our own bodies hold together in this way. Challenge 718

What then is electricity? The answer is simple: *electricity is nothing in particular*. It is the name for a field of inquiry, but not the name for any specific observation or effect. Electricity is neither electric current, nor electric charge, nor electric field. It is not a specific term; it applies to *all* of these phenomena. One has to be a little careful when using it. In fact the vocabulary issue hides a deeper question, which was unanswered in the twentieth century: what is the nature of electric charge? Since charge flows, one can start by asking:

Can one feel the inertia of electricity?

If electric charge really is something *flowing* through metals, one should be able to observe the effects shown in Figure 149, as already Maxwell predicted. Electric charge should fall, have inertia, and be separable from matter. And indeed, each of these effects has been observed.** For example, when a long metal rod is kept vertically, one can measure an electrical potential difference, a voltage, between the top and the bottom. In other words, one can measure the *weight* of electricity this way. Similarly, one can measure potential differences between the ends of an accelerated rod. In particular, one can measure a potential Ref. 372

Challenge 717 * By the way, are batteries sourcesof charges?

** Maxwell tried to detect these effects (apart from the last one, which he did not predict), but his apparatuses where not sensitive enough.

difference between the centre and the rim of a rotating metal disk. This latter experiment was in fact the way in which the ratio q/m for currents in metals was first measured with precision. The result is

$$q/m = 1.8 \cdot 10^{11}\ \mathrm{C/kg} \qquad (331)$$

for all metals, with small variations. In short, electrical current has mass. Therefore, whenever one switches on an electrical current, one gets a *recoil*. This simple effect can easily be measured and confirms the mass to charge ratio just given. Also the emission of current into air or into vacuum is observed; in fact, every television tube uses this principle to generate the beam producing the picture. It works best when metal objects have a sharp, pointed tip. The rays created this way – one could say that they are 'free' electricity – are called *cathode rays*. Within a few percent, they show the same mass to charge ratio as expression (331). This correspondence thus shows that charges in metals move almost as freely as in air; that is the reason metals are such good conductors.

Ref. 373

Ref. 374

If electric charge falls inside vertical metal rode, one can take the astonishing deduction that cathode rays – as we will see later, they consist of free electrons* – should not be able to fall through a vertical metal tube. This is due to the fact that the electrical field generated by the displaced electricity in the tube precisely compensates the acceleration of gravity, so that electrons should not be able to fall through long thin cylinders.The experiment has indeed been performed, and a reduction of the acceleration of free fall for electrons of 90% has been observed. Can you imagine why the ideal value of 100% is not achieved?

Challenge 719

Ref. 375

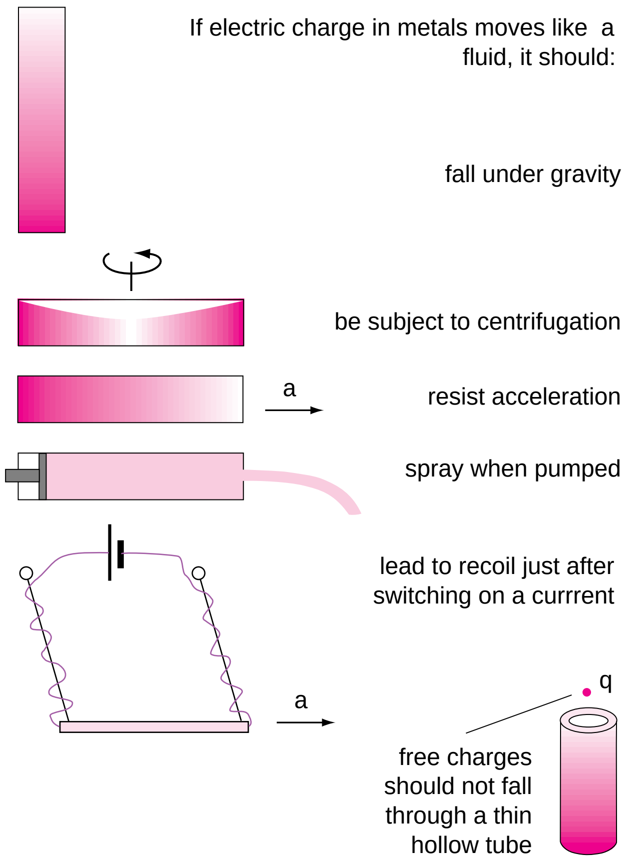

Figure 149 Consequences of the flow of electricity

How fast do charges move?

In vacuum, such as inside a colour television, charges accelerated by a tension of 30 kV

* The name 'electron' is due to Johnstone Stoney. Electrons are the smallest and lightest charges moving in metals; they are, usually, the 'atoms' of electricity. Their charge is small, 0.16 aC, so that flows of charge typical of everyday life consist of large numbers of electrons; as a result, they behave like a continuous fluid.

move with a third of the speed of light.In modern particle accelerators charges move so rapidly that their speed is indistinguishable from that of light for all practical purposes. Challenge 720

In metals, electric signals move roughly with speeds around the speed of light. (Actually, the precise value depends on the capacity of the cable, and is usually in the range $0.3c$ to $0.5c$.) This is due to the ability of metals to easily take in arriving charges and to let depart others. But when one measures the speed of charges inside metals, the *electrons*, one gets the same value as for ketchup inside its bottle, namely around $1\,\mathrm{mm/s}$. Are you able to explain this apparent contradiction? Challenge 721

Inside liquids, charges move with different speed than inside metals, and their charge to mass ratio is also different. We all know that from direct experience. Our *nerves* work by using electric signals and take (only) a few milliseconds to respond to stimuli, even though they are metres long. A similar speed is observed inside semiconductors and inside batteries. In all these systems, moving charge is transported by *ions*; they are charged atoms. Ions, like atoms, are large and composed entities, in contrast to the tiny electrons.

In other systems, charges move both as electrons and as ions. Examples are neon lamps, fire, plasmas, or the sun itself. Inside atoms, electrons behave even more strangely. One tends to think that they turn around the nucleus (as we will see later) at rather high speed, as the orbit is so small. However, it turns out that in most atoms many electrons do not turn around the nucleus at all. The strange story behind atoms and their structure will be told in the second part of this mountain ascent.

How can one make a motor?

Communism is soviets plus electricity.
Lenin (1870, Simbirsk-1924, Gorki)

The reason for this famous statement were two discoveries. One was made in 1820 by the Danish physicist Hans Christian Oersted (1777–1851) and the other in 1831 by the English physicist Michael Faraday.* The consequences of these experiments changed the world completely in less than one century.

On the 21st of July of 1821, Oersted published a leaflet, in Latin, which took Europe by storm. Oersted had found that when a current is sent through a wire, a nearby magnet is put into motion. In other words, he found that the flow of electricity can move bodies.

Further experiments show that *two* wires in which charges flow attract or repel each other, depending on whether the currents are parallel or antiparallel. These and other experiments

* Michael Faraday (1791, Newington, Surrey–1867) born in a simple family, without schooling, of deep and simple religious ideas, as a boy he became assistant of the most famous chemist of his time, Humphry Davy. Without mathematical training, at the end of his life he became member of the Royal society. A modest man, he refused all other honours in his life. He worked on chemical topics, the atomic structure of matter, and most of all, developed through all his experimental discoveries, such as induction, paramagnetism, diamagnetism, electrochemistry, the Faraday effect, and the idea of (magnetic) field and field lines. Fields were later described mathematically by Maxwell, who at his time was the only one in Europe who took over the concept.

show that wires in which electricity flows behave the like magnets.* In other words, Oersted had found that electricity could be turned into magnetism.

Shortly afterwards, Ampère** found that *coils* increase these effects dramatically. Coils behave like little magnets. In particular, coils, like magnetic fields, have always two poles, usually called the north and the south pole. Opposite poles attract, similar poles repel each other. As is well known, the earth is itself a large magnet, with its magnetic north pole near the geographic south pole, and vice versa.

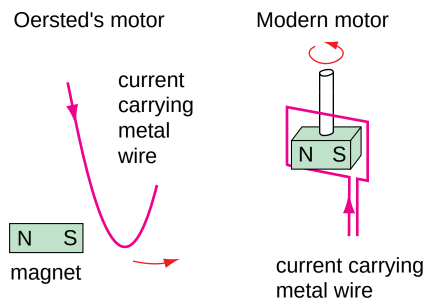

Figure 150 An ancient and a modern version of an electric motor

Experiments show that the magnetic field turns out to always have a given direction in space, and to have a magnitude common to all (resting) observers. One is tempted to describe it by a vector. However, this is wrong, since a magnetic field does not behave like an arrow when placed before a mirror. It turns out that a magnetic field pointing towards a mirror does not change direction for the mirror set up. Are you able to confirm this using what was told about magnetic fields up to now?

Challenge 722

In other words, it is not completely correct to describe a magnetic field by a vector $\mathbf{B} = (B_x, B_y, B_z)$; the precise way is to describe it by the quantity***

$$\mathrm{B} = \begin{pmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{pmatrix} , \qquad (332)$$

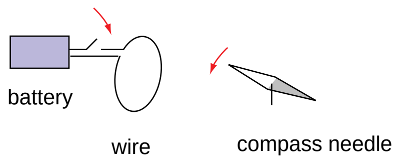

Figure 151 An electrical current always produces a magnetic field

called an *antisymmetric tensor*. (It is also called a *pseudovector*; note that also angular momentum and torque are examples of such quantities.) In summary, *magnetic fields* are defined by the acceleration

∗ In fact, if one imagines tiny currents moving in circles inside magnets, one has the same description for all magnetic fields observed in nature.

∗∗ André-Marie Ampère (1775, Lyon–1836, Marseille), French physicist and mathematician. Autodidact, he read the famous *encyclopédie* as a child; in a life full of personal tragedies, he wandered from maths to chemistry and physics, worked as a high school teacher, and published nothing of importance until 1820. Then the discovery of Oersted reached all of Europe: electrical current can deviate magnetic needles. Ampère worked for years on the problem, and published in 1826 the summary of his findings, which lead Maxwell to call him the Newton of electricity. He named and developed many parts of electrodynamics. The unit of electrical current is named after him.

∗∗∗ The quantity B was not called 'magnetic field' until recently. We follow here the modern, logical definition, which is superseding the traditional one, in which B was called the 'magnetic flux density' or 'magnetic induction' and a different quantity, **H**, was called – incorrectly – the magnetic field. That quantity will not appear in this walk, but is important for the description of magnetism in materials.

they impart on moving charges. This acceleration turns out to follow

$$\mathbf{a} = \frac{e}{m}\mathbf{v}\mathrm{B} = \frac{e}{m}\mathbf{v}\times\mathbf{B} \tag{333}$$

a relation which is often called *Lorentz acceleration* after the important Dutch physicist Hendrik A. Lorentz (Arnhem, 1853–Haarlem, 1928) who first stated it clearly.* The Lorentz acceleration is the effect at the basis of any electric motor. An electric motor is a device using magnetic fields as efficiently as possible to accelerate charges flowing in a wire. Through their motion the wire is then moved as well, and thus electricity is transformed into motion.

Like in the electric case, we now need to know how the *strength* of magnetic fields is determined. Experiments like Oersted's show that the magnetic field is due to moving charges, and that a charge moving with velocity **v** produces a field given by

$$\mathrm{B}(\mathbf{r}) = \frac{\mu_o}{4\pi} q \frac{\mathbf{v}\times\mathbf{r}}{r^3} \quad \text{where} \quad \frac{\mu_o}{4\pi} = 10^{-7}\,\mathrm{N/A^2} \tag{334}$$

Again, the strange factor $\mu_o/4\pi$ is due to the historical way the electrical units were defined. It is easy to see that the field has an intensity given by $\mathbf{vE}/c^2$, where **E** is the electric field measured by an observer moving *with* the charge.It looks as if magnetism is a relativistic effect.** Challenge 724

In 1831, Michael Faraday discovered an additional piece of the puzzle. He found that a moving magnet could cause a current flow in an electrical circuit. Magnetism can thus also be turned into electricity. This important discovery allowed the production of electrical current flow with generators, so-called *dynamos*, using water power, wind power or steam power. They starting the modern use of electricity in our world. Behind every electrical plug there is a dynamo somewhere.

Additional experiments show that magnetic fields also lead to electric fields when one changes to a moving viewpoint, as you might check on any of the examples of the Figures 150 to 154. *Magnetism indeed is relativistic electricity*. Electric and magnetic fields are partly transformed into each other when switching from one inertial reference frame to the other. Magnetic and electrical fields thus behave like space and time, which are also mixed up when changing from one inertial frame to the other. The theory of special relativity then tells us that there must be a single concept, an *electromagnetic field*, describing them both. Investigating the details, one finds that the electromagnetic field F surrounding charged bodies has to be described by an antisymmetric 4-*tensor*

$$\mathrm{F}^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \quad \text{or} \quad \mathrm{F}_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{335}$$

Obviously, the electromagnetic field, and thus every component of these matrices, depends on space and time. The matrices show that electricity and magnetism are two faces of the

Challenge 723 * Does the definition of magnetic field given here assume a speed of the charge much smaller than that of light?

Challenge 725 ** Equation (334) is valid only for small velocities and accelerations. Can you find the general one?

same effect; in addition, since electric fields appear only in the topmost row and leftmost column, the expressions show that in everyday life, for small speeds, electricity and magnetism *can* be separated.

Actually, the expression for the field contains everywhere the expression $1/\sqrt{\mu_o \varepsilon_o}$ instead of the speed of light c. We will explain the reason for this substitution shortly.

The total influence of electric and magnetic fields on fixed or moving charges is then given by the following expression:

$$
\begin{aligned}
& m\mathrm{b} = \mathrm{F}u \quad \text{or} \\
& m\frac{du^{\mu}}{d\tau} = q\mathrm{F}^{\mu}{}_{\nu}u^{\nu} \quad \text{or} \\
& m\frac{d}{d\tau}\begin{pmatrix} \gamma c \\ \gamma v_x \\ \gamma v_y \\ \gamma v_z \end{pmatrix} = q\begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & B_y & -B_x & 0 \end{pmatrix}\begin{pmatrix} \gamma c \\ \gamma v_x \\ \gamma v_y \\ \gamma v_z \end{pmatrix} \quad \text{or} \\
& W = q\mathbf{E}\mathbf{v} \quad \text{and} \quad d\mathbf{p}/dt = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})
\end{aligned} \tag{336}
$$

All four expressions describe the same content; the simplicity of the first one is the reason for the involved matrices (335) of the electromagnetic field. In fact, the extended *Lorentz relation* (336) is the *definition* of the electromagnetic field, since the field is defined as that 'stuff' which accelerates charges. In particular, all devices which put charges into motion, such as batteries and dynamos, as well as all devices which are put into motion by flowing charges, such as electric motors and muscles, are described by this relation. That is why it is usually studied already in high school. The Lorentz relation describes all cases in which the motion of objects can be seen by the naked eye or felt by our senses, such as the movement of electrical motors in high speed trains, in elevators and in dental drills, the motion of the picture generating electron beam in television tubes, or the travelling of electrical signals in cables and in the nerves of the body. Ref. 376, 377

The electromagnetic field tensor F is an *antisymmetric* 4-tensor. (Can you write down the relation between $\mathrm{F}^{\mu\nu}$, $\mathrm{F}_{\mu\nu}$, and $\mathrm{F}^{\mu}{}_{\nu}$?)Like any such tensor, it has two invariants, i.e., two properties which are the same for every observer: the expression $B^2 - E^2/c^2 = \frac{1}{2}\mathrm{tr}\,\mathrm{F}^2$ and the product $4\mathbf{EB} = -c\;\mathrm{tr}\,\mathrm{F}^{*}\mathrm{F}$. (Can you confirm this?) Challenge 726 Challenge 727

The first expression turns out to be the Lagrangian of the electromagnetic field. It is a scalar and implies that if E is larger, smaller, or equal cB for one observer, it also is for all other observers. The second invariant, a pseudoscalar, describes whether the angle between the electric and the magnetic field is acute or obtuse for all observers.*

* There is in fact a third Lorentz invariant, much less known. It is specific to the electromagnetic field and is a combination of the field and its vector potential:

$$
\begin{aligned}
\kappa_3 &= \frac{1}{2}A_{\mu}A^{\mu}\mathrm{F}_{\rho\nu}\mathrm{F}^{\nu\rho} - 2A_{\rho}\mathrm{F}^{\rho\nu}\mathrm{F}_{\nu\mu}A^{\mu} \\
&= (\mathbf{AE})^2 + (\mathbf{AB})^2 - |\mathbf{A}\times\mathbf{E}|^2 - |\mathbf{A}\times\mathbf{B}|^2 + 4A^4(\mathbf{AE}\times\mathbf{B}) - A^8(E^2+B^2)
\end{aligned} \tag{337}
$$

Ref. 378 This expression is Lorentz (but not gauge) invariant; knowing it can help clarify unclear issues, such as the lack of existence of waves in which the electric and the magnetic field are parallel. Indeed, for plane monochromatic

The application of electromagnetic effects to daily life has opened up a whole new world which did not exist before. Electrical light, electric motors, radio, telephone, X-rays, television, and computers changed human life completely in less than one century. For example, the installation of electric lighting in city streets has almost eliminated the previously so common night assaults. These and all other electrical devices use the fact that charges can flow in metals, that one can translate electromagnetic energy into mechanical energy (sound, motors), into light (lamps), into heat and coldness (ovens, refrigerators), that one can send electromagnetic fields across the air (radio and television, remote controls), and that one can use electric or magnetic fields to store information (computers).

How motors prove relativity right

> The only mathematical operation I performed in my life
> was to turn the handle of a calculator.
> Michael Faraday

All electric motors are based on the result that electric currents interact with magnetic fields. The simplest example is the attraction of two wires carrying parallel currents. This observation alone, made in 1820 by Ampère, is sufficient to make motion larger than a certain maximal speed impossible. Ref. 379

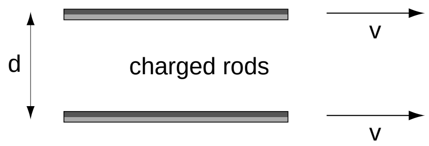

Figure 152 The relativistic aspect of magnetism

The argument is beautifully simple. We change the original experiment and imagine two long, electrically charged rods of mass m, moving in the same direction with velocity v and separation d. An observer moving with the rods would see an electrostatic repulsion between the rods given by Challenge 729

$$ma_e = -\frac{1}{4\pi\varepsilon_o}\frac{2\lambda^2}{d} \qquad (338)$$

where λ is the charge per length of the rods. A second, resting observer sees two effects: the electrostatic repulsion and the attraction discovered by Ampère. He therefore observes finds Challenge 730

$$ma_{em} = -\frac{1}{4\pi\varepsilon_o}\frac{2\lambda^2}{d} + \frac{\mu_o}{2\pi}\frac{\lambda^2 v^2}{d} \qquad (339)$$

It is easy to check that the second observer sees a repulsion, as the first one does, only if

$$v^2 < \frac{1}{\varepsilon_o\mu_o} \quad . \qquad (340)$$

This maximum speed, with a value of $0.3\,\mathrm{GM/s}$, is thus valid for any object carrying charges. But all everyday objects contain charges: there is thus a maximum speed for matter. Are you able to expand the argument to neutral particles as well?More on this limit velocity, Challenge 731

waves all three invariants *vanish* in the Lorentz gauge. Also the quantities $\partial_\mu J^\mu$, $J_\mu A^\mu$ and $\partial_\mu A^\mu$ are Lorentz invariants. (Why?)The latter, the frame independence of the divergence of the four potential, reflects the invariance of gauge choice. The gauge in which the expression is set to zero is called the *Lorentz gauge*. Challenge 728 which we know already, will be found out below.

In summary, electric effects are due to flow of electric charges and to electric fields. Magnetism is due to *moving* electric charges. It is *not* due to magnetic charges.* The strength of magnetism, used in any running electric motor, proves relativity right: there is a maximum speed in nature. Both electric and magnetic fields carry energy and momentum. They are two faces of the same coin. However, our description of electromagnetism is not complete yet; we need the final description of the way charges *produce* the electromagnetic field.

The description of electromagnetic field evolution

In the years between 1861 and 1865, pondering the details of all experiments known to him, James Clerk Maxwell produced a description of electromagnetism which forms one of the pillars of physics.** Maxwell took all experimental results and extracted their common basic principles, shown in Figures 153 and 154. Twenty years later, Heaviside and Hertz extracted the main points of Maxwell ideas and called the summary *Maxwell's theory of the electromagnetic field*. It consists of two equations (four in the nonrelativistic case).

See page 873

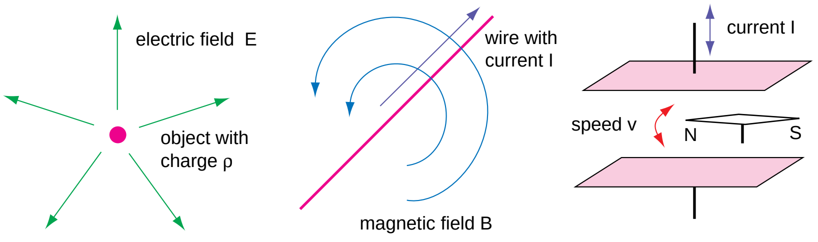

Figure 153 The first of Maxwell's equations

* 'Electrons move in metal with a speed of about 1 mm; thus if I walk along a cable carrying a constant current with the same speed, I should not be able to sense any magnetic field.' What is wrong with theargument?

Challenge 732

** James Clerk Maxwell (1831, Edinburgh–1879, Cambridge), scottish physicist; founded electromagnetism by unifying electricity and magnetism theoretically, as described in this chapter. His work on thermodynamics forms a second pillar of his activity. In addition, he also studied the theory of colours and developed the now standard horseshoe colour diagram; he was one of the first persons to make a colour photograph. He is often seen as the greatest physicist ever. Clerk and Maxwell were both his family names.

The first result is the precise description for the fact that electromagnetic fields *originate at charges*, and nowhere else. The corresponding equation is variously written*

$$
\begin{aligned}
&d\mathrm{F} = j\sqrt{\frac{\mu_o}{\varepsilon_o}} \quad \text{or} \\
&d^{\nu}\mathrm{F}_{\mu\nu} = j^{\mu}\sqrt{\frac{\mu_o}{\varepsilon_o}} \quad \text{or} \qquad\qquad (341)\\
&(\partial_t/c, -\partial_x, -\partial_y, -\partial_z)\begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} = \sqrt{\frac{\mu_o}{\varepsilon_o}}(\rho, j_x/c, j_y/c, j_z/c) \quad \text{or} \\
&\nabla\mathbf{E} = \frac{\rho}{\varepsilon_o} \quad \text{and} \quad \nabla\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_o\mathbf{j} \quad ,
\end{aligned}
$$

putting into many signs a simple statement: *electrical charge carries the electromagnetic field*. This statement, including its equations, are equivalent to the three basic observations of Figure 153. It describes Coulomb's relation, Ampère's relation, and the way changing currents induce magnetic effects, as you may want to check. Challenge 733

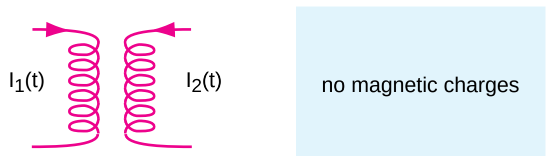

Figure 154 The second of Maxwell's equations

The second result by Maxwell is the precise description of how changing electric fields create magnetic fields, and vice versa. In particular, the electric field can have vortices only when there is a changing magnetic field. In addition it expresses the observation that in nature there are no magnetic charges, i.e. that magnetic fields have no sources. All these results are described by the the relation variously written

$$
\begin{aligned}
&d\,{}^*\mathrm{F} = 0 \quad \text{with} \quad {}^*\mathrm{F}^{\rho\sigma} = \tfrac{1}{2}\varepsilon^{\rho\sigma\mu\nu}\mathrm{F}_{\mu\nu} \quad \text{or} \\
&\varepsilon_{\mu\nu\rho}\partial_\mu\mathrm{F}_{\nu\rho} = \partial_\mu\mathrm{F}_{\nu\rho} + \partial_\nu\mathrm{F}_{\rho\mu} + \partial_\rho\mathrm{F}_{\mu\nu} = 0 \quad \text{or} \\
&\begin{pmatrix} \gamma\frac{1}{c}\partial_t \\ \gamma\partial_x \\ \gamma\partial_y \\ \gamma\partial_z \end{pmatrix}\begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z/c & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & E_x/c & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \qquad\qquad (342)\\
&\nabla\mathbf{B} = 0 \quad \text{and} \quad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad .
\end{aligned}
$$

The relation expresses the *lack of sources for the dual field tensor*, usually written ${}^*\mathrm{F}$. There are no magnetic charges, i.e. no magnetic monopoles in nature. In practice, one always needs this equation together with the previous one. Can you see why? Challenge 734

* Maxwell generalized this equation to cases that the charges are not surrounded by vacuum, but located inside matter. We do not explore these situations in our walk; as we will see during our mountain ascent, the apparently special case of vacuum in fact describes all of nature.

We now have a system as organized as the expression $a = GM/r$ for gravitation. Together with Lorentz' evolution equation (336), which describes how charges move given the motion of the fields, Maxwell's evolution equations (342) and (343) describe *all* electromagnetic phenomena at everyday scales, from portable phones, car batteries, to personal computers, lasers, lightnings, holograms, and rainbows.

We will not study many applications of the equations in our mountain ascent; we continue directly towards our aim to understand the connection to everyday motion and to motion of light. In fact, the electromagnetic field has an important property which we mentioned already right at the beginning: it itself can also move.

The gauge field: the electromagnetic vector potential

The study of moving fields is called *field theory*, and electrodynamics is the major example. (The other classical example is fluid dynamics.) Field theory is a beautiful topic; field lines, equipotential lines, and vortex lines are some of the concepts introduced in this domain. They fascinate many.* However, in this mountain ascent we keep the discussion focussed on motion.

We have seen that fields force us to extend our concept of motion. Motion is not only the state change of objects and of space-time, but also the *state change of fields*. We therefore need, also for fields, a complete and precise description of their state. The observations with amber and magnets have shown us that *fields possess energy and momentum*, which they can impart to particles. The experiments with motors have shown that objects can add energy and momentum to fields. One therefore has to define a *state function* which allows to define energy and momentum for electric and magnetic fields.

Maxwell defined the state function in two standard steps. The first step is the definition of the *(magnetic) vector potential*, which describes the momentum per charge the field
Ref. 380 provides:

$$\mathbf{A} = \frac{\mathbf{p}}{q} \quad . \tag{343}$$

When a charged particle moves through a magnetic potential $\mathbf{A}(\mathbf{x})$, its momentum changes by $q\mathbf{A}$. Due to this definition, the vector potential has the property that

$$\mathrm{B} = \nabla \times \mathbf{A} = \mathrm{curl}\,\mathbf{A} \tag{344}$$

i.e. that the magnetic field is the curl of the magnetic potential.** For example, the vector
Challenge 736 potential for a long straight current carrying wire is parallel to the wire, and has the value

$$A(r) = -\frac{\mu_o I}{2\pi}\ln(r) \tag{345}$$

Challenge 735 * What is the relation, for static fields, between field lines and (equi-) potential surfaces?Can a field line cross a potential surface twice?

For more details, see the *free* textbook by Bo Thidé, *Electromagnetic Field Theory*, on his
Ref. 368 http://www.plasma.uu.se/CED/Book web site. And of course, in English, the texts by Schwinger and by
Ref. 376 Jackson.

** The curl is called the *rotation* and abbreviated rot in most languages.

depending on the distance r from the wire. For a solenoid, the vector potential 'circulates' around it. Inside the solenoid, the vector potential increases from the centre. Similarly, for a constant and uniform magnetic field B one finds the vector potential

$$\mathbf{A}(\mathrm{r}) = -\frac{1}{2}\mathbf{B}\times\mathbf{r} \quad . \tag{346}$$

However, there is a catch. The magnetic potential is *not* defined uniquely. If $\mathbf{A}(\mathbf{x})$ is a vector potential, then

$$\mathbf{A}'(\mathbf{x}) = \mathbf{A}(\mathbf{x}) + \operatorname{grad}\Lambda \tag{347}$$

where $\Lambda(t,\mathbf{x})$ is some scalar function, is also a vector potential for the same situation. The magnetic field B stays the same, though. What happens to the corresponding momentum values? They also change.

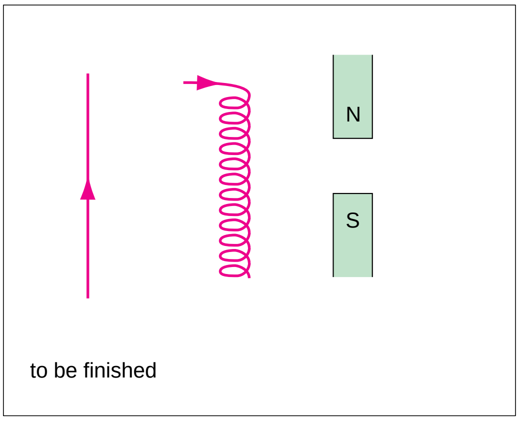

Figure 155 Vector potentials for selected situations

One is more accustomed to the fact that like momentum, also the energy of the electromagnetic field is defined ambiguously. Indeed, the second step in the specification of a state for the electromagnetic field is definition of the *electric potential* as the energy U per charge: Ref. 380

$$\varphi = \frac{U}{q} \tag{348}$$

In other words, the potential $\varphi(\mathbf{x})$ at a point $\mathbf{x}$ is the energy needed to move a unit charge to the point $\mathbf{x}$ starting from a point where the potential vanishes. The potential energy is thus given by $q\varphi$. Due to this definition, the electric field $\mathbf{E}$ is simply the *change* of the potential with position corrected by the time dependence of momentum, i.e.

$$\mathbf{E} = -\nabla\varphi - \frac{\partial}{\partial t}\mathbf{A} \quad , \tag{349}$$

Obviously, there is a freedom in the choice of the definition of the potential. If $\varphi(\mathbf{x})$ is a possible potential, then

$$\varphi'(\mathbf{x}) = \varphi(\mathbf{x}) - \frac{\partial}{\partial t}\blacksquare \tag{350}$$

is also a potential function for the same situation. This freedom is the generalization of the fact that energy is defined only up to a constant. Nevertheless, the electric field $\mathbf{E}$ remains the same for all potentials.

In relativistic 4-vector notation, the state function of the electromagnetic field becomes

$$A^{\mu} = (\varphi/c, \mathbf{A}) \quad . \tag{351}$$

It is easy to see that it is a complete description of the field, since one has

$$\mathrm{F} = dA \quad \text{or} \quad \mathrm{F}^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{352}$$

which means that the electromagnetic field is completely specified by the 4-potential A. But as just said, the 4-potential itself is *not* uniquely defined. Indeed, any other gauge field A' related to A by the *gauge transformation*

$$A'^{\mu} = A^{\mu} + \partial^{\mu}\Lambda \tag{353}$$

where $\Lambda = \Lambda(t,x)$ is any arbitrarily chosen scalar field, leads to the *same* electromagnetic field, and to the same accelerations and evolutions. The gauge 4-field A is thus an *overdescription* of the physical situation as several *different* A correspond to the *same* physical situation. Therefore one has to ensure that all measurement results are independent of gauge transformations, i.e. that all observables are gauge invariant quantities. Such gauge invariant quantities are, as we just saw, the fields F and *F, and in general all classical quantities. We note that many theoretical physicists use the term 'electromagnetic field' for the quantity A_{μ}.

There is a simple image, due to Maxwell, to help overcoming the conceptual difficulties of the vector potential. It turns out that the closed line integral over A_{μ} is gauge invariant,
Challenge 737 because

$$\oint A_{\mu} dx^{\mu} = \oint (A_{\mu} + \partial_{\mu}\Lambda) dx^{\mu} = \oint A'_{\mu} dx^{\mu} \quad . \tag{354}$$

In other words, if one pictures the vector potential as a quantity allowing to associate a number to a tiny ring at each point in space, one gets a good, gauge invariant picture of the vector potential.*

Now that we have defined a state function which describes energy and momentum, let us see what happens in more detail when electromagnetic fields move.

Colliding charged particles

A simple experiment clarifies the just defined properties of electromagnetic fields. When two charged particles collide, one finds that their total momentum is *not* conserved.

Imagine two particles of identical mass and charge just after a collision, when they move away from each other. Imagine also that the two masses are large, so that the acceleration due to their electrical repulsion is small. For an observer in the centre of gravity of the two, each particle feels an acceleration from the electrical field of the other, given by
Challenge 738 the so-called *Heaviside formula*

$$E = \frac{q(1 - v^2/c^2)}{4\pi e_o r^2} \quad . \tag{355}$$

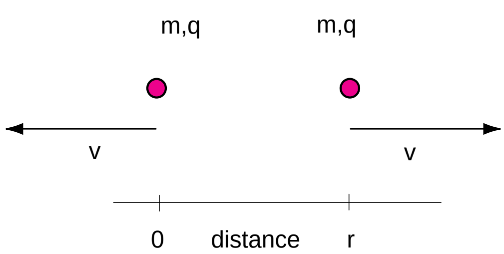

Figure 156 Charged particles after a collision

In other words, the total system has a vanishing total momentum.

Ref. 381 * In the part on quantum mechanics we will see that the exponent of this expression, namely $\exp(iq \oint A_{\mu} dx^{\mu})$, usually called the *phase factor*, can indeed be directly observed in experiments.

Take a second observer, moving with respect to the first with velocity v, so hat the first charge will be at rest. The expression of the electrical field leads to two different values for the electric fields at the position of each particle. In other words, the system of the two particles is not in inertial motion, as we would expect; the total momentum is not conserved. Where did it go? Ref. 382 Challenge 739

This at first surprising effect has even been put in form of a theorem, by Van Dam and Wigner. They showed that for a system of particles interacting at a distance the total energy-momentum cannot remain constant in all inertial frames. Ref. 383

The total momentum of the system is conserved only because the electromagnetic field itself also carries momentum. In other words, electromagnetic fields are able to *hit* objects and to be hit by them. As we will show below, also light is an electromagnetic field. Thus one should be able to move objects by shining light onto them. One should even be able to suspend particles in mid air by shining light onto them from below. Both predictions are correct, and a few experiments describing them will be presented shortly.

One concludes that any sort of field leading to particle interactions must carry energy and momentum, as the argument applies to all such cases. In particular, it applies to the nuclear interactions. Indeed, in the second part of our mountain ascent we will even find an additional result: all fields are themselves composed of particles. The energy and momentum of fields then become an obvious state of affairs.

The Lagrangian of electromagnetism

The motion of charged particles and the motion of the electromagnetic field can also be described using a Lagrangian instead of using the three equations given above. It is not hard to see that the actionS_{CED} for a particle in classical electrodynamics can be symbolically definedby* Challenge 740

$$S_{\mathrm{CED}} = -mc^2 \int d\tau - \frac{1}{4\mu_o} \int \mathrm{F} \wedge * \mathrm{F} - \int j \wedge A \qquad (356)$$

which in index notation becomes

$$S_{\mathrm{CED}} = -mc \int_{-\infty}^{\infty} \sqrt{\eta_{\mu\nu} \frac{dx_n^{\mu}(s)}{ds} \frac{dx_n^{\nu}(s)}{ds}}\, ds - \int_{\mathbf{M}} \left(\frac{1}{4\mu_o} \mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} + j_{\mu} A^{\mu} \right) d^4x \quad .$$

In other words, the least action principle still states that the change of a system is always as small as possible. New is the measure of the change produced by the electromagnetic field. Its internal change is given by the term FF, and the change due to interaction with matter is given by the term jA.

* The symbol $\wedge$, 'wedge', in fact has a precise mathematical meaning; but its background, the concept of *(mathematical) form*, carries us too far from our walk. An electrodynamics text completely written with forms is Kurt Meetz & Walter L. Engl, *Elektromagnetische Felder – Mathematische und physikalische Grundlagen*, Springer, 1980.

The action S_{CED} leads to the evolution equations by requiring that it be stationary under variations δ, δ' of the positions and of the fields which vanish at infinity, i.e. that

$$\begin{aligned} \delta S = 0 \quad & \text{when} \quad x_\mu = x_\mu + \delta_\mu \quad \text{and} \quad A_\mu = A_\mu + \delta'_\mu \quad , \\ & \text{provided} \quad \delta x_\mu(\theta) \rightarrow 0 \quad \text{for} \quad |\theta| \rightarrow \infty \\ & \text{and} \quad \delta A_\mu(x_\nu) \rightarrow 0 \quad \text{for} \quad |x_\nu| \rightarrow \infty \quad . \end{aligned} \tag{357}$$

See page 121
Challenge 741

In the same way as in the case of mechanics, using the variational method for the two variables A and x, one recovers the evolution equations for particle and fields

$$b^\mu = \frac{q}{m} \mathrm{F}^\mu_\nu u^\nu \quad , \quad \partial_\mu \mathrm{F}^{\mu\nu} = j^\nu \sqrt{\frac{\mu_o}{\varepsilon_o}} \quad , \quad \text{and} \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu \mathrm{F}_{\rho\sigma} = 0 \tag{358}$$

which we know already. Obviously, they are equivalent to the variational principle based on S_{CED}. Both descriptions have to be completed by specifying *initial conditions* for the particles and the fields, as well as *boundary conditions* for the latter. One needs the first and zeroth derivatives of the position of the particles, and the zeroth derivative for the electromagnetic field.

Challenge 742

Are you able to specify the Lagrangian of the pure electrodynamic field using the fields **E** and **B**?

Challenge 743

Note that this result also implies that electromagnetism is *time reversible*. That means that every example of motion due to electric or magnetic causes can also take place backwards. This is easily deduced from the properties of the Lagrangian. On the other hand, everyday life shows many electric and magnetic effects which are not time invariant, such as electromagnetic breaking, electric light bulbs, etc. Can you explain how this fits together?

In summary, with Lagrangian (356) all of classical electrodynamics is described and understood. For the rest of this chapter, we look at some specific topics from this vast field.

Symmetries: the energy-momentum tensor

We know from classical mechanics that we get the definition of energy and momentum tensor by using Noether's theorem, if we determine the conserved quantity from the Lorentz symmetry of the Lagrangian. For example, we fond that relativistic particles have an energy-momentum *vector*. At the point at which the particle is, it describes the energy and momentum.

Since the electromagnetic field is not a localized entity like a point particle, but extended, one needs to know the *flow* of energy and momentum at each point, separately *for each direction*. This makes a description with a *tensor* necessary.

– CS – to be continued – CS –

In summary, electrodynamic motion, like all other examples of motion encountered so far, is deterministic, is conserved, and is reversible. No big news. But two special symmetries of electromagnetism deserve special mention.

What is a mirror?

We will study the strange properties of mirrors several times during our walk. We start with the simplest one first. Everybody can observe, by painting his hands in two different colours, that a mirror does *not* exchange right and left, as little as it exchanges up and down; however, a mirror does exchange right and left *handedness*. In fact, it does so by exchanging front and back.

Electrodynamics give a second answer: a mirror is a device that switches magnetic north and south poles. Can you confirm this? Challenge 744

But is it always possible to distinguish left from right? This seems easy: this text is rather different from a bɘɿoɿɿim version, as are many other objects in our surroundings. But take a simple landscape. Are you able to say which of the two pictures of Figure 157 is the original?

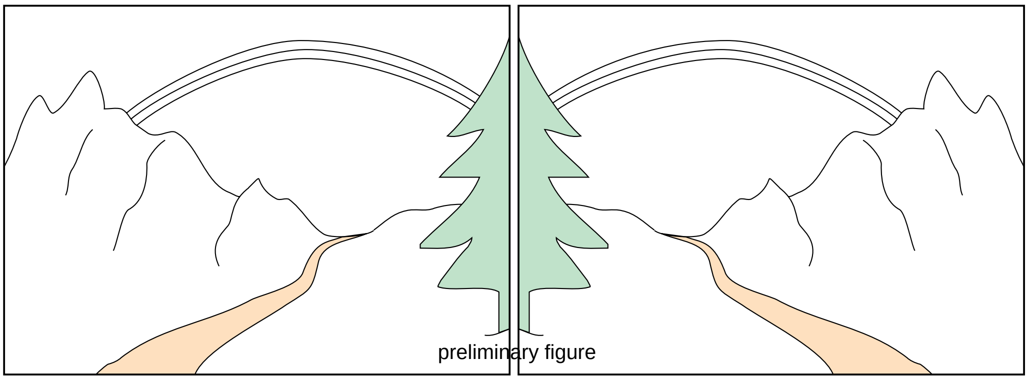

Figure 157 Which one is the original landscape?

Astonishingly, it is actually impossible to distinguish a picture of nature from its mirror image if it does not contain any human traces. In other words, everyday nature is somehow left-right symmetric. This observation is so common that all candidate exceptions, from the jaw movement of ruminating cows to the helical growth of plants, have been studied See page 645 extensively.* Can you name a few more? Challenge 745

The left-right symmetry of nature is a consequence of the fact that everyday nature is described by gravitation and, as we will see, by electromagnetism. Both interactions share an important property: by substituting all coordinates in their equations by the negative of their values the equations remain unchanged. This means that for any solution of these equations, i.e. for any naturally occurring system, the mirror image is also a possibility which can occur naturally. Everyday nature thus cannot distinguish between right and left.

* The most famous is the position of the heart. The mechanisms leading to this disposition are still being investigated. Most recent research is suggesting that the oriented motion of the cilia on embryos, probably Ref. 384 in the region called *node*, determine the right left asymmetry. The deep origin of this asymmetry is not yet elucidated, however.

Another asymmetry of the human body is the hair whirl on the back of the head; the majority of humans having only one, and in 80% of the cases it is left turning.

Indeed, there are right *and* left handers, people with their heart on the left *and* others with their heart on the right side, etc.

To explore further this strange aspect of nature, try the following experiment: imagine you are exchanging radio messages with a martian; are you able to explain him what right and left are, so that when you will meet, you are sure you are talking about the same thing? Challenge 746

Actually, the mirror symmetry of everyday nature – also called its *parity invariance* – is so pervasive that most animals cannot even distinguish left from right in a deeper sense. Ref. 385 Most animals react to mirror stimuli with mirror responses. It is hard to teach them different ways to react, and it is possible almost only for mammals. The many experiments performed on this topic gave the result that animals have symmetrical nervous systems, and possibly only humans show *lateralization*, i.e. a preferred hand and a different use for the left and the right part of the brain.

To sum up this digression, classical electrodynamics is left-right symmetric, or parity invariant. Can you show this using its Lagrangian? Challenge 747

What is the difference between electric and magnetic fields?

Obviously, the standard answer is that electric fields have sources, and magnetic fields do not; that moreover magnetic fields are small relativistic effects of importance only when charge velocities are high or when electrical fields cancel out.

For situations with matter, this clear distinction is correct. Up to the present day, no particle with a magnetic charge, called a *magnetic monopole*, has ever been found, even though its existence is possible in several unified models of nature. See page 648 If found, the action (356) would have to be modified by the addition of a fourth term, namely the magnetic current density. However, no such particle has yet been detected, despite intensive search efforts.

But in vacuum, when matter is not around, it is possible to take a completely different view. In vacuum the electric and the magnetic field can be seen as two faces of the same quantity, since a transformation such as

$$\begin{aligned} \mathbf{E} &\to c\mathbf{B} \\ \mathbf{B} &\to -\mathbf{E}/c \end{aligned} \tag{359}$$

called (electromagnetic) *duality* transformation, transforms each vacuum Maxwell equation into the other. (In fact, there are even more such transformations; can you spot them?)Alternatively, the duality transformation transforms F into *F. In other words, in vacuum one *cannot* distinguish electric from magnetic fields. Challenge 748

Matter would be symmetric under duality only if magnetic charges, also called magnetic monopoles, would exist. In that case the transformation (359) could be extended to

$$c\rho_e \to \rho_m \quad , \quad \rho_m \to -c\rho_e \quad . \tag{360}$$

It was one of the great discoveries of theoretical physics that even though classical electrodynamics with matter is not symmetric under duality, nature is. In 1977, Claus Montonen and David Olive showed that quantum theory allows duality transformations even *with* the inclusion of matter. It was already known since the 1930s that quantum theory allows magnetic monopoles. We will discover the important ramifications of this result in the third part of the text. This duality turns out to be one of the essential stepping stones leading to a unified description of motion. (A somewhat difficult question: Extending this duality to quantum theory, can you deduce what transformation is found for the fine structure constant, and why it is so interesting?) See page 660 Challenge 749

Duality, by the way, is a symmetry that works *only* in Minkowski space-time, i.e. in space-times of 3 plus 1 dimensions. Mathematically, it is closely related to the existence of quaternions, to the possibility of interpreting Lorentz boosts as rotations in 3+1 dimensions, and last but not least, to the possibility to define other smooth mathematical structures than the standard one on the space R^4. These mathematical connections are still somewhat mysterious at the time being; they somehow point to the special role that four space-time dimensions play in nature. More details will become clear in the third part of our mountain ascent.

14. What is light?

An important consequence of the equations of electrodynamics was deduced by Maxwell in 1865. He found that in the case of vacuum, the equations of the electrodynamic field could be written as

$$\Box \mathbf{A} = 0 \quad \text{or} \quad \varepsilon_o \mu_o \frac{\partial^2 \varphi}{dt^2} + \frac{\partial^2 A_x}{dx^2} + \frac{\partial^2 A_y}{dy^2} + \frac{\partial^2 A_z}{dz^2} = 0 \quad . \tag{361}$$

This is called a *wave equation*, because it admits solutions of the type Challenge 750

$$\mathbf{A}(t,x) = \mathbf{A}_0 \sin(\omega t - \mathbf{k}\mathbf{x} + \delta) \tag{362}$$

which are commonly called *(plane) waves*. Such a wave satisfies equation (361) for any value of the *amplitude* A_0, of the *phase* δ, and of the *angular frequency* ω, provided the *wave vector* $\mathbf{k}$ satisfies the relation

$$\omega(\mathbf{k}) = \frac{1}{\sqrt{\varepsilon_o \mu_o}} k \quad \text{or} \quad \omega(\mathbf{k}) = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \sqrt{\mathbf{k}^2} \tag{363}$$

The relation $\omega(\mathbf{k})$ between the angular frequency and the wave vector, the so-called *dispersion relation*, is the main property of any type of wave, be it a sound wave, a water wave, an electromagnetic wave, or any other kind. Relation (363) specifically characterizes electromagnetic waves in vacuum, and distinguishes them from all other types of waves.*

Equation (361) for the electromagnetic field is *linear* in the field; this means that the sum of two situations allowed by it is itself an allowed situation. Mathematically speaking, any *superposition* of two solutions is a solution as well. For example, this means that two waves can travel through each other without disturbing each other, and that waves can travel through static electromagnetic fields. Linearity also means that any electromagnetic

* Just to be complete, a *wave* in physics is any propagating imbalance. Other types of waves, such as sound, water waves, earthquakes, etc., will not be studied much in this mountain ascent.

wave can be described as a superposition of pure sine waves, each of which is described by expression (362).

After Maxwell had predicted the existence of electromagnetic waves, in the years between 1885 and 1889 Heinrich Hertz* discovered and studied them, by fabricating a very simple transmitter and receiver for 2 GHz waves. Waves around this frequency are used in the last generation of mobile telephones. These waves are now called *radio waves*, since physicists tend to call all moving force fields *radiation*, recycling an old term which originally meant 'light emission.'

Hertz also measured the speed of these waves; today everybody can do that by himself by telephoning to a friend on another continent using a satellite line (just use a low cost provider). There is about half a second additional delay between the end of a sentence and the answer of the friend, compared to normal conversation. In this half second, the signal goes up to the geostationary satellite, down again and back. This gives a speed of $c \approx 4 \cdot 36\,000\,\mathrm{km}/0.5\,\mathrm{s} \approx 3 \cdot 10^5\,\mathrm{km/s}$, which is close to the precise value.

But Maxwell did more. He also predicted that *light* itself is a solution of equation (362) and therefore an electromagnetic wave, albeit with a much higher frequency. This famous prediction can be checked in many ways.

It is easy to confirm the *wave* properties of light, and indeed they were known already long before Maxwell. In fact, the first to suggest that light is a wave was, around the year 1678, the important Dutch physicist Christiaan Huygens (1629, 's Gravenhage –1695, Hofwyck). One can confirm this fact with one's own fingers. Simply put your hand one or two centimetres from the eye, look towards the sky through the gap between the middle finger and the index, and let the two fingers almost touch. You will see a number of dark lines dividing the gap. These lines are the interference pattern formed by the light behind the slit created by the fingers. *Interference* is the name given to those amplitude patterns which appear when several waves superpose.** This experiment therefore also allows to estimate the wavelength

Challenge 752 of light, and thus if you know its speed, also its frequency. Are you able to do so?

Historically, a similar effect was central in convincing everybody that light was a wave: the supernumerary rainbows, the additional bows below the main rainbow. If one looks carefully at a rainbow, below the main reed yellow green blue violet bow, one observes

Ref. 386 weaker, additional green blue and violet bows. Depending on the intensity of the rainbow, several of these supernumerary rainbows can be observed. They are due to an interference

See page 386 effect, as Thomas Young showed around 1803.*** (More about the rainbow below.) It seems

* Heinrich Rudolf Hertz (1857, Hamburg–1894, Bonn), important hamburger theoretical and experimental physicist. The unit of frequency is named after him. Despite his early death, Hertz was a central figure in the development of electromagnetism, in the explanation of Maxwell's theory, and in the unfolding of radio communication technology. More about him on page 109.

Challenge 751 ** Where does the energy go in interference patterns?

*** Thomas Young (1773, Milverton–1829), read the bible at two, spoke Latin at four; doctor of medicine, he became professor of physics. He introduced the concept of *interference* into optics, explaining the Newtonian rings and rainbow, and was the first person to determine light's *wavelength*, a concept that he also introduced, and its dependence on colour. He was the first to deduce the three colour vision explanation of the eye and after reading of the discovery of polarization, explained light as a transverse wave. In short he discovered most what people learn at high school about light. He was a universal talent: he also worked on the deciphering of hieroglyphs, on ship building, and on engineering problems. In Britain his ideas on light were not accepted, that in those times scientists either did not trust their own fingers, or did not have any.

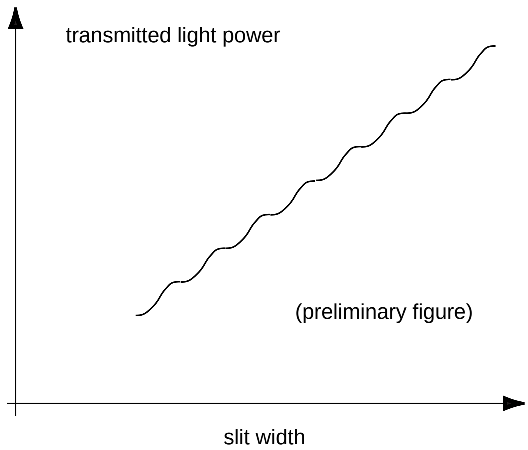

Figure 158 The light power transmitted through a slit as function of its width

There are many other ways that the wave character of light becomes apparent. Maybe the most beautiful is an experiment carried out by a team of Dutch physicists in 1990. They simply measured the light transmitted through a *slit* in a metal plate. It turns out that the transmitted intensity depends on the width of the slit. Their surprising result is shown in Figure 158. Can you explain the origin of the unexpected steps in the curve? Ref. 387 Challenge 753

Numerous other experiments on the creation, detection, and measurement of electromagnetic waves have been performed in the nineteenth and twentieth century. For example, in 1800, William Herschel discovered infrared using a prism and a thermometer. (Can you guess how?)A bit later, Johann Wilhelm Ritter, a colourful figure of natural Romanticism, discovered ultraviolet light, using solver chloride, AgCl, and a prism again. The result of all these experiments is that electromagnetic waves can be distinguished first of all by their frequency or wavelength. The main categories are listed in Table 37. Challenge 754 See page 380

At the end of the twentieth century the final confirmation has become possible. Using quite sophisticated experiments it became possible to measure the oscillation frequency of light directly. The value, between 375 and 750 THz, is so high that detection was impossible for many years. But with these modern experiments the dispersion relation (363) of light has finally been confirmed completely. Ref. 388

So far, we avoided one question about light. If light oscillates, in which direction does this happen? The answer is hidden in the parameter $\mathbf{A}_0$ in expression (362). Electromagnetic waves oscillate in directions *perpendicular* to their motion. Therefore, even for identical frequency and phase, waves can still differ: they can have different *polarization* directions. For example, the polarization of radio transmitters determine whether radio antennas of receivers have to be kept horizontal or vertical. Also for light, polarization is easily achieved, e.g. by shining it through stretched plastic films. When the polarization of light was discovered in 1808 by the French physicist Louis Malus, it definitively established its wave nature. Malus discovered it when he looked at the strange double images produced by feldspat, a transparent crystal found in many minerals. Feldspat splits light beams into two, it is *birefringent*, and polarizes them differently. That is the reason that feldspat is part of every crystal collection. If you ever see a piece of feldspat, have a look at it.

since Newton's followers crushed all opposing views. Young collaborated with Fraunhofer and Fresnel; at last, his results were made famous by Fresnel and Helmholtz.

By the way, the human eye is unable to detect polarization, in contrast to many insects. As is well known honey bees use polarization to deduce the position of the sun even when Ref. 389 it is hidden behind clouds, and many insects use polarization to distinguish water surfaces Challenge 755 from mirages. Can you find out how?On the other hand, both the cornea and the lens of the Ref. 390 human eye are birefringent.

Note that all possible polarizations of light form a continuous set. However, a general wave can be seen as a superposition of two orthogonal, linearly polarized waves with different amplitudes and different phases. Most books show pictures of plane, linearized electrodynamic waves. Essentially, the electric fields look like water waves generalized to three dimensions, the same for magnetic fields, and the two are perpendicular to each other. Can Challenge 756 you confirm this?

Interestingly, a generally polarized plane wave can also be seen as the superposition of right and left *circularly polarized waves*. However, no figures of such waves are found in Challenge 757 any textbook. Can you explain why?

So far it is clear that light is a wave. To confirm that the nature of light is indeed *electromagnetic* is more difficult. The first argument was by Maxwell. From equation (363), he deduced a prediction for the speed of electromagnetic waves, namely the celebrated expression

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{364}$$

Challenge 758 which you should be able to confirm.When Maxwell inserted the values in the right hand side, he found, within measurement errors, complete correspondence with the measured speed of light. Note that the right hand side contains electric and magnetic quantities, and the left hand side is an optical entity. The expression thus unifies electromagnetism with optics.

Of course, people were not yet completely convinced. They looked for more ways to show that light is electromagnetic in nature. Since Maxwell's evolution equations are linear, electric or magnetic fields alone do not influence the motion of light. On the other hand, since electromagnetic waves are emitted only by accelerated charges, and since all light is emitted from matter, one follows that matter is full of electromagnetic fields and accelerated electric charges. This implies that the influence of matter on light could be understood from its internal electromagnetic fields, and in particular, that subjecting matter to *external* electromagnetic fields should change its the light it emits, the way it interacts with light, or generally, the material properties as a whole.

For example, it is indeed found that electric fields can influence the light transmission of oil, an effect discovered by Kerr. With time, many more influences of matter in fields on light were found, and a more extensive list is given in the table on page 407. It turns out that with a few exceptions the effects can *all* be described by the electromagnetic Lagrangian S_{CED} (356), or equivalently, by Maxwell's equations (358). In summary, classical electrodynamics indeed unifies the description of electricity, of magnetism, and of optics; all phenomena in these fields, from the rainbow to radio, from lightnings to electric motors, are found to be different aspects of the evolution of the electromagnetic field F.

Table 37 The electromagnetic spectrum

Frequency	Wave-length	Name	Main properties	Appearance	Use
$3 \cdot 10^{-18}$ Hz	10^{26} m	**lower frequency limit**		see section on cosmology	
<10 Hz	>30 Mm	**quasistatic fields**		intergalactic, galactic, stellar, and planetary fields, brain, electrical fish	power transmission, accelerating and deflecting cosmic radiation
		radio waves		electronic devices	
10 Hz-50 kHz	30 Mm-6 km	ELW	go round the globe, penetrate into water	nerve cells, electromechanical devices	power transmission, communication with submarines www.vlf.it
50 -500 kHz	6 km-0.6 km	LW	follow earth curvature, felt by nerves (‘bad weather nerves’)	emitted by thunderstorms	radio communications, telegraphy, inductive heating
500 - 1500 kHz	600 m-200 m	MW	reflected by night sky		radio
1.5 -30 MHz	200 m-10 m	SW	circle world if reflected by the ionosphere, destroy hot air balloons	emitted by stars	radio transmissions, radio amateurs, spying
15 -150 MHz	20 m-2 m	VHF	allow battery operated transmitters		remote controls, closed networks, tv, radio amateurs, radio navigation, military, police, taxi
150 - 1500 MHz	2 m-0.2 m	UHF	idem, line of sight propagation		radio, walkie-talkies, tv, cellular phones, internet via cable, satellite communication, bicycle speedometers
		microwaves			
1.5 -15 GHz	20 cm-2 cm	SHF	idem, absorbed by water	night sky, emitted by hydrogen atoms	radio astronomy, used for cooking (2.45 GHz), telecommunications, radar
15 -150 GHz	20 mm-2 mm	EHF	idem, absorbed by water		
		infrared	go through clouds	emitted by every warm object	satellite photography of earth, astronomy
3 -100 THz	1000 -3 μm	IRC or far infrared		sunlight	
100 -210 THz	3 μm-1.4 μm	IRB or medium infrared		sunlight	used for optical fibre communications for telephone and cable TV

Frequency	Wave-length	Name	Main properties	Appearance	Use
210 -385 THz	1400-780 nm	IRA or near infrared	penetrates for several cm into human skin	sunlight, radiation from hot bodies	healing of wounds, rheumatism, sport physiotherapy, hidden illumination
375 -750 THz	800-400 nm	**light**	not absorbed by air, detected by the eye (up to 850 nm at sufficient power)	heat (‘hot light’), lasers & chemical reactions e.g. phosphor oxidation, fireflies (‘cold light’)	definition of straightness, enhancing photosynthesis in agriculture, photodynamic therapy, hyperbilirubinaemia treatment
375 -478 THz	780-627 nm	red	penetrate flesh	blood	alarm signal, used for breast imaging
	700 nm	pure red		rainbow	colour reference for printing, painting, illumination and displays
478 -509 THz	627-589 nm	orange		various fruit	attracts birds and insects
	600 nm	standard orange			
509 -530 THz	589-566 nm	yellow		majority of flowers	idem; best background fro reading black text
	580 nm	standard yellow			
530 -606 THz	566-495 nm	green	maximum eye sensitivity	algae and plants	highest brightness per light energy to the human eye
	546.1 nm	pure green		rainbow	colour reference
606 -688 THz	495-436 nm	blue		sky, gems, water	
	488 nm	standard cyan			
	435.8 nm	pure blue		rainbow	colour reference
688 -789 THz	436-380 nm	indigo, violet		flowers, gems	
		ultraviolet			
789 -952 THz	380-315 nm	UVA	penetrate ca. 1 mm into skin, darken it, produce vitamin D, suppress immune system, cause skin cancer, destroy eye lens	emitted by sun and stars	seen by certain birds, integrated circuit fabrication
0.95 - 1.07 PHz	315-280 nm	UVB	idem, destroy DNA, cause skin cancer	idem	idem

Frequency	Wave-length	Name	Main properties	Appearance	Use
1.07 -3.0 PHz	280-100 nm	UVC	form oxygen radicals from air, kill bacteria, penetrate ca. 10 μm into skin	idem	disinfection, water purification, waste disposal, integrated circuit fabrication
3 -24 PHz	100-13 nm	EUV			sky maps, silicon lithography
		X-rays	penetrate materials	emitted by stars, plasmas, and black holes	imaging human tissue
24 -240 PHz	13-1.3 nm	soft X-rays	idem	synchrotron radiation	idem
$>$ 240 PHz or $>$ 1 keV	$<$ 1.2 nm	hard X-rays	idem	emitted when fast electrons hit matter	crystallography, structure determination
$>$ 12 EHz or $>$ 50 keV	$<$ 24 pm	**γ-rays**	idem	radioactivity, cosmic rays	chemical analysis, disinfection, astronomy
$1.9 \cdot 10^{43}$ Hz	$\approx 10^{-35}$ m	**Planck limit**		see part three of this text	

The expression of the speed of light does not depend on the proper motion of the observer measuring the electromagnetic fields involved. This strange result was the first hint that the speed of light is a universal constant. However, it took several decades before the consequences we realized and relativity was developed.

As a note, it is often told that the teenager Albert Einstein asked himself what would happen if an observer would move at the speed of light, and in particular, what kind of electromagnetic field he would observe. He once explained that this Gedankenexperiment convinced him already at that young age that *nothing* could travel at the speed of light, since the field observed would have a property not found in nature. Can you guess which one? Challenge 759

Does light travel straight?

Usually this is the case, since we even use light to *define* 'straightness.' However, there are a few exceptions and every expert on motion should know them. Ref. 391

In sugar syrup, light beams curve, as shown in Figure 159. In fact, light bends at any material interface. This effect, called *refraction*, is the same that makes aquaria seem less deep than they actually are. Refraction is a consequence of the change of light speed from material to material. Are you able to explain refraction, and thus explain the syrup effect? Challenge 760

Refraction in water droplets is also the basis of the rainbow, as shown on page 386, and refraction in ice crystals in the atmosphere is at the basis of the halos and the many other light patterns often seen around the sun and the moon. Ref. 392

A second important observation is that light goes around corners, and the more so the more they are sharp. This effect is called *diffraction* and is also due to the wave nature of light. You probably remember it from high school. In fact, light goes around corners in the same way that sound does.

Because of diffraction, it is impossible to produce strictly parallel light beams. For example, every laser beam diverges by a certain minimum amount, called the *diffraction limit*. Maybe you know that the world's most expensive cat-eye is on the moon, where it has been deposited by the Apollo 11 cosmonauts. Can you determine how wide a laser beam with minimum divergence has become when it arrives at the moon, assuming that it was 1 m wide when sent to the moon? How wide would it come back if it had been 1 mm wide at the start?

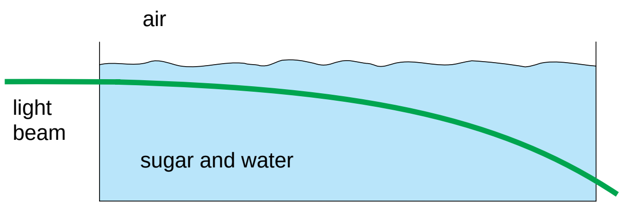

Figure 159 Sugar water bends light

Ref. 393

Challenge 761

Diffraction implies that there are no perfectly sharp images: there exists a *limit on resolution*. This is true for the eye as well, where the resolution is between one and two minutes of arc, i.e. between 0.3 and 0.6 mrad. The limit is due to the limited size of the pupil. Therefore for example, there is a maximum distance at which one can distinguish the two headlights of a car. Can you estimate it?

Challenge 762

For the same reason it is impossible to see the Great Wall in northern China from the moon, contrary to what is often claimed. In the few parts which are not yet in ruins, the wall is about 6 metres wide, and even if it casts a wide shadow during the morning or the evening, the angle it subtends is way below a second of arc, so that it is completely invisible to the human eye. In fact, three different cosmonauts who went to the moon performed careful searches and confirmed that the claim is absurd. The story is one of the most tenacious urban legends. (Is it possible to see the wall from the space shuttle?)In fact the largest man-made objects are the polders of reclaimed land in the Netherlands; they *are* visible from outer space. So are most large cities as well as the highways in Belgium at night; their bright illumination makes them stand out clearly from the dark side of the earth.

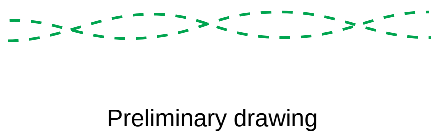

Figure 160 Light beams can spiral around each other

Ref. 394

Challenge 763

Diffraction also means that behind a small disk illuminated along its axis, the centre of the shadow shows, against all expectations, a bright spot. This spot was predicted in 1819 by Denis Poisson in order to show to what absurd consequences the wave theory of light would lead. He had just read the mathematical description of diffraction developed by Augustin Fresnel on the basis of the wave description of light. But shortly afterwards, François Arago* actually observed Poisson's point, making Fresnel famous, and the wave properties of light started to be generally accepted.

* François Arago (1786–1853), French physicist. Augustin Jean Fresnel (1788–1827), engineer and part time physicist; he published in 1818 his great paper on wave theory for which he got the price of the French academy

Electromagnetic fields do not influence light directly, since light has no charge, and since Maxwell's equation are linear. But in some materials the equations are non-linear, and the story changes. For example, in certain photorefractive materials, two nearby light beams can even *twist* around each other, as shown by Segev and coworkers in 1997. Ref. 395

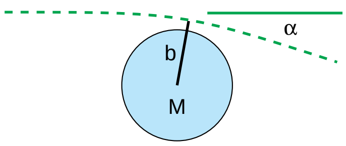

Figure 161 Masses bend light

A final way to bend light is gravity, as discussed already in the chapters on universal gravity and on general relativity. Also the effect of gravity between two light beams was discussed there. See page 320

In summary, light travels straight only if it travels *far from other matter*. In everyday life, 'far' simply means more than a few millimetres, because all gravitational and electromagnetic effects are negligible at these distances, mainly due to lights' truly supersonic speed.

Can one touch light?

If one takes a little glass bead and poses it on top of a powerful laser, the bead remains suspended in mid air, as shown in Figure 162. That means that light has momentum. Therefore, contrary to what we said in the beginning of our mountain ascent, images *can* be touched! In fact, the ease with which objects can be pushed has even a special name. For stars, it is called the *albedo,* and for general objects it is called the *reflectivity* r.

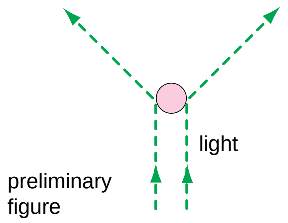

Figure 162 Levitating a small glass bead with a laser

Like each type of electromagnetic field, and like every kind of wave, light carries energy; the energy flow per surface and time is Challenge 764

$$\mathbf{P} = \frac{1}{\mu_o}\mathbf{E}\times\mathbf{B} \quad \text{giving an average} \quad <P>= \frac{1}{2\mu_o}E_{max}B_{max} \quad . \tag{365}$$

Obviously, light also has momentum. It is related to the energy by

$$p = \frac{E}{c} \tag{366}$$

As a result, the pressure p exerted by light onto a body is given by Challenge 765

$$p = \frac{P}{c}(1+r) \tag{367}$$

where for black bodies one has $r = 0$ and for mirrors $r = 1$; other bodies have values in between. What is your guess for the amount of pressure due to sunlight on a black surface of one square metre?Is that the reason that we feel more pressure during the day than during the night? Challenge 766

of sciences in 1819. To improve his finances, he worked in the commission responsible for lighthouses, for which he developed the well-known Fresnel lens. He died prematurely, partly of exhaustion due to overwork.

In fact, one needs rather delicate equipment to detect the momentum of light, or if one prefers, its radiation pressure. In order to achieve this, in 1873, William Crookes * invented the *light mill radiometer*. He had the intention to demonstrate the radiation pressure of light. The light mill consists of four thin plates, black on one side and shiny on the other, which are mounted on a vertical axis, as shown in Figure 163. However, when Crookes finished building it – it was similar to those sold in shops today – he found, like everybody else, that it turned in the wrong direction, namely with the shiny side towards the light! (Why is it wrong?)You can check it by yourself by pointing a laser pointer onto it. The behaviour has been a puzzle for quite some time. Explaining it involves the tiny amount of gas left over in the glass bulb and takes us too far from the topic of our mountain ascent. Only in 1901, with the advent of much better pumps, it was possible to create a sufficiently good vacuum that allowed to measure the light pressure with such an improved, true radiometer.

Ref. 396

Challenge 767

Ref. 397

Ref. 398

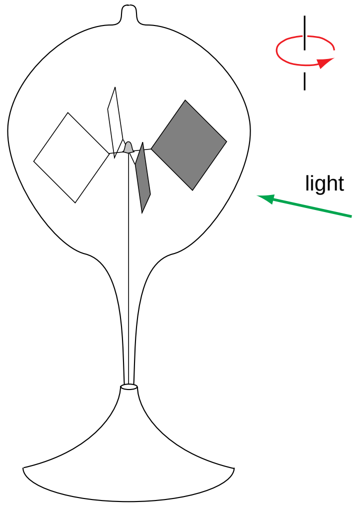

Figure 163 A commercial light mill turns *against* the light

In fact, it turns out that the tail of a comet exists only because the light of the sun hits the small dust particles which detach from the comet. For the same reason, the tail always points *away* from the sun, a well-known fact that you might want to check at the next opportunity.

Challenge 768

But light cannot only touch and be touched, it can also *grab*. In the 1980s, Arthur Ashkin and his research group developed actual *optical tweezers* which allow to grab, to suspend, and to move small transparent spheres of 1 to 20 μm diameter using laser beams. It is possible to do this through a microscope, so that one can also observe at the same time what one is doing. This technique is now routinely used in biological research around the world, and has been used for example to measure the force of single muscle fibres, by chemically attaching their ends to glass or teflon spheres and then pulling them apart with such an optical tweezer.

Ref. 399

But that is not all. In the last decade of the twentieth century, several groups even managed to *rotate* objects, thus realizing actual *optical spanners*. They are able to rotate particles at will in one direction or the other, by changing the optical properties of the laser beam used to trap the particle.

Ref. 399

In fact, it does not take much to deduce that if light has linear momentum, circularly polarized light also has *angular* momentum. In fact, for such a wave the angular momentum L is given by

$$L = \frac{E_{\text{nergy}}}{\omega} \quad . \tag{368}$$

Equivalently, the angular momentum of a wave is $\lambda/2\pi$ times its linear momentum.For light, this result has been confirmed already in the early 20th century: a light beam can put certain

Challenge 769

Ref. 400

* William Crookes (1832, London–1919, London), English chemist and physicist, president of the Royal Society, discoverer of Thallium.

materials (which ones?)into rotation, as shown in Figure 164. Of course, the whole thing works even better with a laser beam. In the 1960s, a beautiful demonstration was performed with microwaves. A circularly polarized microwave beam from a maser – the microwave equivalent of a laser – can put a metal piece absorbing it into rotation. For a beam with cylindrical symmetry, depending on the sense of rotation, the angular momentum is either parallel or antiparallel to the direction of propagation. All these experiments confirm that light also carries angular momentum, an effect which will play an important role in the second part of our mountain ascent. Challenge 770

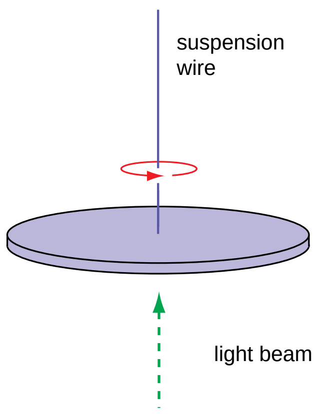

Figure 164 Light can rotate objects

We note that not for all waves angular momentum is energy per angular frequency. This is only the case for waves made of what in quantum theory will be called spin 1 particles. For example, for gravity waves the angular momentum is *twice* this value, and they are therefore expected to be made of spin 2 particles.

In summary, light can touch and be touched. Obviously, if light can rotate, it can also *be* rotated. Could you imagine how this can be achieved? Challenge 771

War, light, and lies

From the tiny effects of the equation (367) for light pressure one deduces that light is not an efficient tool for hitting objects. However, light is able to heat up objects, as one can feel on the skin if it is touched by a laser beam of about 100 mW or more. For the same reason even cheap laser pointers are dangerous to the eye.

In the 1980s, and again in 2001, a group of people who read too many science fiction novels managed to persuade the military – who also indulge in this habit – that lasers could be used to shoot down missiles, and that a lot of tax money should be spent to develop such lasers. Using the definition of the Poynting vector and a hitting time of about 0.1 s, are you able to estimate the weight and size of the battery necessary for such a device to work?What would happen in cloudy or rainy weather? Challenge 772

Other people tried to persuade NASA to study the possibility to propel a rocket using light instead of ejected gas. Are you able to estimate whether this is feasible? Challenge 773

What is colour?

We saw that radio waves of certain frequencies are visible. Within that range, different frequencies correspond to different colours. (Are you able to convince a friend about this?)But the story is not finished here. Numerous colours can be produced either by a single wavelength, i.e. by *monochromatic* light, or by a *mixture* of several other, different colours. For example, standard yellow can be, if it is pure, a beam of 600 nm, or it can be a mixture of standard green of 546.1 nm and standard red of 700 nm. The eye cannot distinguish between the two cases. In everyday life, all colours are mixed, with the exception of those of yellow Challenge 774

street lamps, of most laser beams, and of the rainbow. You can check this yourself, using an
Challenge 775 umbrella or a compact disk.

In particular, white light is a mixture of a continuous range of colours with a given intensity per wavelength. To check that white light is a mixture of colours, simply hold Figure 165 so near to your eye that you cannot focus the stripes any more. The unsharp borders of the white stripes have a pink or a green shade. These colours are due to the imperfections of the lens in the human eye, its so-called *chromatic aberrations*. Aberrations have the consequence that not all light frequencies follow the same path in the lens of the eye, and therefore that they hit the retina at different spots. This is the same effect that occurs in prisms or in water drops showing a rainbow. By the way, the shape of the rainbow tells
Challenge 776 something about the shape of the water droplets. Can you deduce the connection?

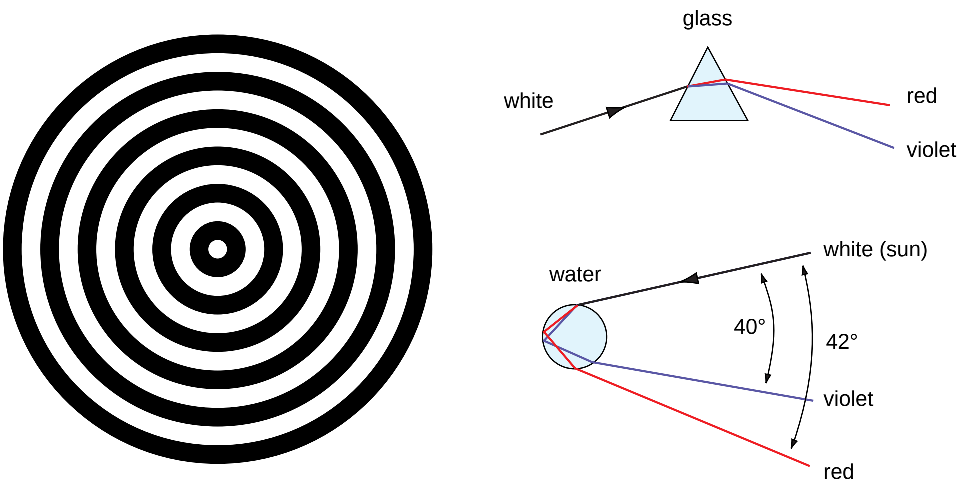

Figure 165 Proving that white light is a mixture of colours

Even pure air splits white light. This is the reason that the sky or far away mountains are blue and that the sun is red at sunset and at dawn. You can repeat this effect by looking
Challenge 777 through water at a black surface or at a lamp.Adding a few drops of milk to the water makes the lamp yellow and then red, and makes the black surface blue (like the sky seen from the earth as compared to the sky seen from the moon). More milk increases the effect. For the same reason, sunsets are especially red after volcanic eruptions.

By the way, at sunset the atmosphere itself acts as a prism as well; that means that the sun is split into different images, one for each colour, which are slightly shifted against each other, a bit like a giant rainbow in which not only the rim, but the whole disk is coloured. The total shift is about 1/60th of the diameter. If the weather is favourable,and if the air is clear and quiet up to and beyond the horizon, for a few seconds it is possible to see, after the red, orange and yellow images have set, the rim of the green-blue image of the sun. That
Ref. 401 is the famous ‘rayon vert’ described by Jules Verne in his novel of the same title. It is often

seen from islands, such as Hawaii.*

To clarify the difference between colours in physics and colour in human perception and language, a famous discovery deserves to be mentioned: colours in language have a natural *order*. (Colours which point to objects, such as aubergine or sepia, or colours which are not generally applicable, such as blond, are excluded in this discussion.) Colours are ordered by all people in the following order: 1st black and white, 2nd red, 3rd green and yellow, 4th blue, 5th brown; 6th come mauve, pink, orange, grey and sometimes a twelfth term different from language to language. The result states that if a particular language has a word for any of these colours, it also has a word for all the preceding ones. The result also implies that people use these basic colour classes even if their language does *not* have a word for each of them. These strong statements have been confirmed for over 100 languages. Ref. 402

What is the speed of light? – Again

Physics is talking about motion. Talking is the exchange of sound; and sound is an example of a signal. A *(physical) signal* is the transport of information using transport of energy. There are no signals without motion of energy. This is also obvious from the fact that there is no way to store information without storing energy. To any signal one can thus ascribe a propagation speed. The highest possible signal speed is also the maximal velocity of general influences, or, using sloppy language, the maximal velocity with which effects spread causes.

If the signal is carried by matter, such as by the written text in a letter, the signal velocity is then the velocity of the material carrier, and experiments show that it is limited by the speed of light.

For a wave carrier, such as water waves, sound, light or radio waves, the situation is less evident. What is the speed of a wave? The first answer that comes to mind is the speed with which wave crests of a sine wave move. This already introduced *phase velocity* is given by the ratio between the frequency and the wavelength of a monochromatic wave, i.e. by

$$v_{\mathrm{ph}} = \frac{\omega}{k} \quad . \tag{369}$$

For example, the phase velocity determines interference phenomena. Light in vacuum has the same phase velocity $v_{\mathrm{ph}} = c$ for all frequencies. Are you able to imagine an experiment to test this to high precision? Challenge 778

On the other hand, there are cases where the phase velocity is larger than c, most notably when light travels through an absorbing substance, and when at the same time the frequency is near to an absorption maximum. In these cases, experiments show that the phase velocity is *not* the signal velocity. For such situations, a better approximation to the signal speed is the *group velocity*, i.e. the velocity at which a group maximum will travel. This velocity is given by Ref. 403

$$v_{\mathrm{gr}} = \left.\frac{d\omega}{dk}\right|_{k_0} \tag{370}$$

* About this and many other topics on colours in nature, such as e.g. the colour of shadows, the halos around the moon and the sun, and many others, see the beautiful book by Marcel Minnaert mentioned on page 58.

where k_0 is the central wavelength of the wave packet. One observes that $\omega = c(k)k = 2\pi v_{ph}/\lambda$ implies the relation

$$v_{gr} = \left.\frac{d\omega}{dk}\right|_{k_0} = v_{ph} + \lambda \frac{dv_{ph}}{d\lambda} \quad . \tag{371}$$

This means that the sign of the last term determines whether the group velocity is larger or smaller than the phase velocity. For a travelling group, as shown by the dotted line in Figure 166, this means that new maxima either appear at the end or at the front of the group. Experiments show that for light *in vacuum*, the group velocity has the same value $v_{gr} = c$ for all values of the wave vector k.

One should be warned however that still many publications propagate the myth that the group velocity *in materials* is never larger than c, the speed of light in vacuum. Actually, the group velocity in materials can be zero, or infinite, or

Challenge 779 evennegative; this happens when the light pulse is very narrow, i.e. when it includes a wide range of frequencies, or again when one is near an absorption transition. In many (but not all) cases the

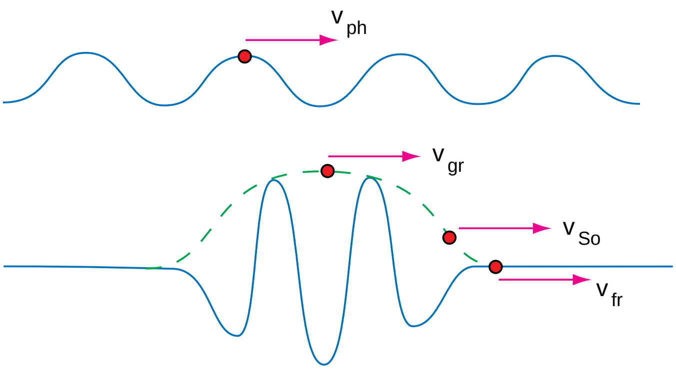

Figure 166 The definition of important velocities in wave phenomena

group is found to widen substantially or even to split, making it difficult to define precisely the group maximum and thus its velocity. Many experiments have confirmed these predictions. For example, the group velocity in certain materials has been measured to be *ten times*

Ref. 404 that of light. The refractive index then is smaller than 1.* However, in all these cases the group velocity is *not* the same as the signal speed.**

What then is the best velocity describing signal propagation? The German physicist Arnold Sommerfeld*** almost solved the main problem in the beginning of the twentieth century. He defined the signal velocity as the velocity v_{So} of the front slope of the pulse, as

Ref. 403 shown in Figure 166. The definition cannot be summarized in a formula, but it does have the property that it describes signal propagation for practically all experiments, in particular

Challenge 780 * Some people have even found $n < o$ for certain microwaves. Can you imagine whatthis means?

** In quantum mechanics, Schrödinger proved that the velocity of an electron is given by the group velocity of its wavefunction. Therefore the same discussion reappeared in quantum theory, as we will find out in the second part of the mountain ascent.

*** Arnold Sommerfeld (1868, Königsberg–1951, München) was a central figure in the spread of special and general relativity, of quantum theory, and of their applications. Professor in Munich, an excellent teacher and text book writer, he worked on atomic theory, on the theory of metals, on electrodynamics, and was the first to understand the importance and the mystery around 'Sommerfeld's famous fine structure constant.'

those in which the group and phase velocity are larger than the speed of light. When studying its properties, one finds that for no material Sommerfeld's signal velocity is larger than the speed of light in vacuum.

One might think that it is conceptually easier to describe signal propagation with help of the energy velocity. As mentioned before, every signal transports energy. The *energy velocity* v_{en} is defined as the ratio between the power flow density $\mathbf{P}$, i.e. the Poynting vector, and the energy density W, both taken in the direction of propagation. For electromagnetic fields – the only ones fast enough to be interesting for eventual superluminal signals – this ratio is

$$\mathbf{v}_{en} = \frac{\mathrm{Re}(\mathbf{P})}{W} = \frac{2c^2\,\mathbf{E}\times\mathbf{B}}{\mathbf{E}^2 + c^2\mathbf{B}^2} \quad . \tag{372}$$

However, like in the case of the front velocity, also in the case of the energy velocity one has to specify if one means the energy transported by the main pulse or by the front. In vacuum, neither is ever larger than the speed of light.* (In general, the energy velocity in matter has a value slightly different from Sommerfeld's signal velocity.) Ref. 403

In recent years, the progress in light detector technology, allowing to detect even the tiniest energies, has forced people to take the fastest of all these energy velocities to describe signal velocity. Using detectors with the highest possible sensitivity one can use as signal the first point of the wave train whose amplitude is different from zero, i.e. the first tiny amount of energy arriving. This point's velocity, conceptually similar to Sommerfeld's signal velocity, is commonly called the *front velocity*, or, to distinguish it even more clearly from Sommerfeld's case, the *forerunner velocity*. It is simply given by Challenge 781

$$v_{fr} = \lim_{\omega\to\infty} \frac{\omega}{k} \quad . \tag{373}$$

The forerunner velocity is *never* larger than the speed of light in vacuum, even in materials. In fact it is precisely c, because for extremely high frequencies, the ratio ω/k is independent of the material, and vacuum properties take over. The forerunner velocity is the true signal velocity, or if one wants, the *true velocity of light*. Using it, all discussions on light speed become clear and unambiguous.

Recently, the issue reappeared in another way. The topic started in 1960 with the 'prediction' by the soviet physicist Victor Veselago that the index of refraction could have *negative* values. In 2000, an experimental 'confirmation' for microwaves was first published. But in Ref. 407 2002 it was shown that negative refraction indices, which imply speeds larger than unity, are only possible for either phase velocity or even group velocity, but not for the energy or true signal velocity. The problems arise because in some physical systems the refraction angle for phase motion and for energy motion differ. If the term 'index of refraction' is consistently used to characterize the motion of energy, it cannot have negative values.

* Signals not only carry energy, they also carry negative entropy ('information'). The entropy of a transmitter increases during transmission. The receiver decreases in entropy (but less than the increase at the transmitter, Ref. 405 of course).

Note that the negative group velocity implies energy transport against the propagation velocity of light. This Ref. 406 is possible only in *energy loaded* materials.

To finish this section, here are two challenges. Which of all the velocities of light is measured in experiments determining the velocity of light, e.g. when light is sent to the
Challenge 782 moon and reflected back?And now a more difficult question: why is the signal speed of
Challenge 783 light slower inside matter, as all experiments show?

Signals and predictions

When somebody reads a text through the phone to a neighbour, who listens to it and maybe repeats it, one speaks of communication. For any third person, the speed of communication is always smaller than the speed of light. But if the neighbour already knows the text, he can say it without waiting to hear the readers' voice. To the third observer such a situation looks like faster than light (superluminal) communication. Prediction can thus *mimic* communication, and in particular, it can mimic faster than light communication. Such a situation has
Ref. 408 been demonstrated most spectacularly in 1994 by Günter Nimtz, who seemingly transported music – all music is predictable for short time scales – through a 'faster-than-light' system. To distinguish between the two situations, one notes that in the case of prediction, no energy transport takes place, in contrast to the case of communication. In other words, the definition of a signal as a transport of information is not as useful and clear-cut as the definition of a signal as *transport of energy*. In the mentioned experiment, no energy was transported faster than light. The same distinction between prediction on one hand and signal or energy propagation on the other hand will be used later on to clarify some famous experiments in quantum mechanics.

> If the rate at which physics papers are being published continues to increase, physics journals will soon be filling library shelves faster than the speed of light. This does not violate relativity since no useful information is being transmitted.

Why can we talk to each other? – Huygens' principle

The properties of our environment often appear in the full importance only when one asks simple questions. Why can we use the radio? Why can we talk on portable phones? Why can we listen to each other? It turns out that a central part of the answer is given by the fact that we live in a space of odd dimensions.

In spaces of even dimensions, one cannot talk, because messages do not stop. This is an important result which is easily checked by throwing a stone into a lake: even after the stone has disappeared, waves are still emitted from the point at which it entered the water. On the contrary, when we stop talking, no waves are emitted any more.

– CS – text to be added – CS –

One can also say that Huygens' principle holds if the wave equation is solved by a circular wave leaving no amplitude behind it. Mathematicians translate this by saying that the delta function $\delta(c^2t^2 - r^2)$ satisfies the wave equation, i.e. that $\partial_t^2\delta = c^2\Delta\delta$. The delta function is that strange 'function' which is zero everywhere except at the origin, where it is infinitely high. A few more properties, not mentioned here, fix the precise way this happens. If one generalizes this to higher dimensions, it turns out that the fundamental solution of the wave equation is zero everywhere only if the space dimension is odd and larger or equal to three.

In summary, when we switch off the light, a room gets dark only because we live in a space of odd dimensions.

How does the world look when riding on a light beam?

This was the question the teenager Albert Einstein tried to answer.* The situation would have strange consequences.

- One would have no mirror image, like a vampire.
- Light would not be oscillating, but a static field.
- Nothing would move, like in the tale of sleeping beauty.

But also at speeds *near* the velocity of light observations would be interesting. One would

- see a lot of light coming towards one and almost no light from behind; the sky would be blue/white in front and red/black in the back;
- observe that everything around happens very very slowly;
- experience the smallest dust particle as deadly bullet.

Can you think of more strange consequences?It is rather reassuring that our planet moves rather slowly through its environment. Challenge 784

Does the aether exist?

Gamma rays, light, and radio waves are moving electromagnetic waves. All exist in empty space. What is oscillating when the light comes along? Maxwell himself called the 'medium' in which this happens the *aether*. The properties of the aether found in experiments are listed in Table 38.

Physical property	experimental value
permeability	$\mu_o = 1.3\ \mu\mathrm{H/m}$
permittivity	$\varepsilon_o = 8.9\ \mathrm{pF/m}$
wave impedance/resistance	$Z_o = 376.7\,\Omega$
conformal invariance	applies
spatial dimensionality	3
topology	R^3
mass and energy content	not detectable
friction on moving bodies	not detectable
own motion	not detectable

Table 38 Experimental properties of the aether and of flat vacuum

The last item of the table is the most important: despite intensive efforts, nobody has been able to detect any *motion* of the aether. In other words, even though the aether oscillates, Ref. 409

* He took the question from a book on the sciences by Aaron Bernstein which he read at that time.

it does not move. Together with the other data, all these results can be summarized in one sentence: there is no way to distinguish the aether from the vacuum: both are oneand the same. Challenge 785

One sometimes hears that relativity or certain experiments show that the aether does not exist. This is incorrect. All the data only show that the aether is indistinguishable from the vacuum. Of course, if one uses the change of curvature as definition for motion of the vacuum, vacuum *can* move, as we will find out in the section on general relativity; but aether still remains indistinguishable from it.*

Later in our mountain ascent we will even find out that the ability of the vacuum to allow the propagation of light and its ability to allow the propagation of particles are equivalent: both require the same properties. Therefore the aether remains *indistinguishable* from vacuum in the rest of our walk. In other words, the aether is a superfluous concept; we drop it from our walk from now on. However, we are not finished with the study of the vacuum; it will keep us busy for a long time, starting with the intermezzo following this chapter. Moreover, quite a few of the aspects in Table 38 will require some amendments later on. Ref. 410

15. Lightning, levitation, and other fun challenges

Electromagnetism and light are almost endless topics. A few points are worth pondering.

▪ How does one wire a light bulb, the mains, and three switches so that the light can be switched on at any of the switches and off at any other switch? And in case of four switches?Nobody will take a physicist serious who can write Maxwell's equations but cannot solve this little problem. Challenge 786

▪ Can you make a mirror that does not exchange left and right? In two different ways? Challenge 787

▪ A scotch tape roll is a dangerous device. Pulling the roll quickly leads to light emission (through triboluminsecence) and to small sparks. It is suspected that several explosions in mines and in ships were triggered when such sparks ignited a combustible gas mixture.

▪ A concave mirror shows an inverted image, if the mirror is bend along the horizontal line. What happens if this mirror is turned around the line of sight? Challenge 788

▪ Electricity produced by friction and by flows of liquids is a small effect. However, in the 1990s, several oil tankers disappeared suddenly, because they had washed their oil tanks by pointing a hose spraying sea water on their walls. The spraying led to charging; with the oil fumes in the tank this led to an explosion and the tankers sank. Similar accidents also happen regularly when chemicals are filled from one tank to another.

▪ When a ship sinks, survivors usually end up in small boats drifting on the sea. Often they are saved by a rope hanging from a helicopter. It is essential a survivor only touches the rope *after* the rope has been in the water, as he can die of heart attack otherwise: the helicopter can be heavily charged.

Ref. 410 * In fact, the term 'aether' has been used as an expression for several different ideas, depending on the author. First of all it was used for the idea that vacuum is not empty, but *full*; secondly, that this fullness can be described by *mechanical models*, such as gears, little spheres, vortices, etc.; thirdly, that vacuum is *similar to matter*, being made of the same substrate. Interestingly, some of these issues will reappear in the third part of our mountain ascent.

▪ The names anode and cathode were suggested by William Whewell and popularized by Michael Faraday. Whewell formed them from the greek; they literally mean ‘upward street’ and ‘descending street’.

▪ The shortest light pulse produced so far had a length of 100 as. How many wavelengths of green light would that correspond to? Challenge 789

▪ Why doe one see shadows of houses, shadow of trees, but never shadows of the electrical cables hanging over streets? Challenge 790

▪ How would you measure the speed of the tip of a lightning? What range do you expect? Challenge 791

▪ How would you show that electrical charge comes in smallest chunks? Challenge 792

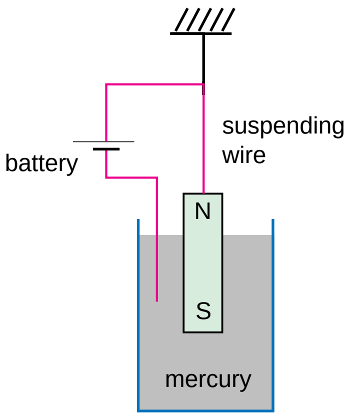

Figure 167 Unipolar motor

▪ One of the simplest possible electric motors was discovered by Faraday in 1831. A magnet suspended into mercury starts to turn around its axis if a current flows through it. In addition, if the magnet is made to turn from outside, the device (in other geometries also called Barlow’s wheel) also works as a current generator, and people even tried to generate domestic current with such a system! Can you explain how it works? Ref. 419 Challenge 793

▪ Cosmic radiation consists of charged particles hitting the earth. (We will discuss it in more detail later.) Astrophysicists explain that these particles are accelerated by the magnetic fields around the galaxy. However, the expression of the Lorentz acceleration shows that magnetic fields can only change the direction of the velocity of charges, not its magnitude. How can one get acceleration nevertheless? See page 636 Ref. 411 Challenge 794

▪ The magnetic field of the earth, much higher than that of other planets because of the moon, with a dipole strength of $7.8 \cdot 10^{22}$ Am^2, shields us from lethal solar wind and cosmic radiation particles. We owe it our life. Ref. 427

▪ The ionosphere around the earth has a resonant frequency of 7 Hz; for this reason any apparatus measuring low frequencies always gets a strong signal at this value. Can you give an explanation of the frequency? Challenge 795

▪ What would be the potential of the earth if one could take all the electrons of a drop of water away? Challenge 796

▪ The sun would be visible to the naked eye only up to a distance of 50 light years. True? Challenge 797

▪ At home, electricity is mostly used as alternating current. In other words, no electron actually flows through cables; as the speed of metal electrons is about 1 mm/s, electrons just move back and forward by 20 μm, Nothing is flowing in or out of the cables! Why do the electricity companies require an actual flow of money in return, instead of being satisfied with a back and forth motion of money? Challenge 798

▪ If one calculates the Poynting vector for a charged up magnet – or simpler, a point charge near a magnet – one gets a surprise: the electromagnetic energy flows in circles around the magnet. Where does this angular momentum come from? Challenge 799

Worse, any atom is an example of such a system – actually of two such systems. Why is this effect not taken into account in calculations in quantum theory? Ref. 412

▪ Ohm's law, the observation that for almost all materials the current is proportional to the voltage, is due to a high school teacher. Georg Simon Ohm explored the question in great depth; at those times, such measurements were difficult to perform.* This has changed now. Recently, even the electrical resistance of single atoms has been measured: in the case of xenon it turned out to be about $10^5\ \Omega$. It was also found that lead atoms are ten times more conductive than gold atoms. Can you imagine why?

Ref. 413
Challenge 800

▪ The charges on two capacitors in series are not generally equal, as naive theory states. For perfect, leak-free capacitors the voltage ratio is given by the inverse capacity ratio $V_1/V_2 = C_2/C_1$, due to the equality of the electric charges stored. However, in practice this is only correct for a few up to a few dozen minutes. Why?

Ref. 414
Challenge 801

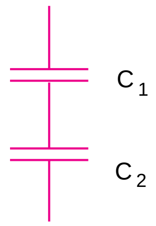

Figure 168 Capacitors in series

▪ Does it make sense to write Maxwell's equations in vacuum? Both electrical and magnetic fields require charges in ordered to be measured. But in vacuum there are no charges! In fact, only quantum theory solves this apparent contradiction. Are yo able to imagine how?

Challenge 802

▪ Grass is usually greener on the other side of the fence. Can you give an explanation based on observations for this statement?

Ref. 415
Challenge 803

▪ Inside a conductor there is no electric field. Thus there is no danger if a lightning hits an aeroplane, as long the plane is made of metal. Aeroplanes are so-called *Faraday cages*. More generally speaking, a field or a charge on the metal surface of a body does not influence fields and charges inside it. Can you give an explanation?

Challenge 804

The explanation will allow you to answer the following question. Are there Faraday cages for gravity as well? Why?

Cars also are good approximations of Faraday cages. If your car is hit by lightning in dry weather, you should wait a few minutes before leaving it, though. Can you imagine why?

Faraday cages also work the other way round. Electric fields changing inside a Faraday cage are not felt outside. For this reason, radios, mobile phones and computers are surrounded by boxes made of metal or metal-sprayed plastics. The metal keeps the so-called *electromagnetic smog* inside buildings to a minimum.

For purely magnetic fields, the situation is more complex. It is quite difficult to shield the inside of a machine from outside magnetic fields. How would you do it? In practice one often uses layers of so-called *mu-metal*; can you guess what this material does?

▪ The *electric polarizability* is the property of matter responsible for the deviation of water flowing from a faucet by a charged comb. It is defined as the strength of electric dipole induced by an applied electric field. The definition simply translates the observation that many objects acquire charges when an electric field is applied. Incidentally, how precisely combs get charged when rubbed, a phenomenon called *electrification*, is still one of the mysteries of modern science.

See page 353

* Georg Simon Ohm (1789, Erlangen–1854, München), bavarian school teacher and physicist. His efforts were recognized only late in his life, and he eventually was promoted to professor at the University in München. Later the unit of *electrical resistance*, the proportionality factor between voltage and current, was named after him.

▪ A pure magnetic field cannot be transformed into a pure electric field by change of observations frame. The best that can happen is that one gets a state similar to an equal mixture of magnetic and electric fields. Can you provide an argument elucidating this relation? Challenge 805

▪ Researchers are trying to detect tooth decay with help of electric currents, using the fact that healthy teeth are bad conductors, in contrast to teeth with decay. How would you make use of this effect in this case? Ref. 416 Challenge 806

▪ A team of camera men in the middle of the Sahara were using battery driven electrical equipment to make sound recordings. Whenever the microphone cable was a few tens of metres long, they also heard a 50 Hz power supply noise, even though the next power supply was thousands of kilometres away. An investigation found that the high voltage lines in Europe loose a considerable amount of power by irradiation; those 50 Hz waves are reflected by the ionosphere around the earth and thus can disturb recording in the middle of the desert. Can you estimated whether this observation implies that living directly near a high voltage line is dangerous? Challenge 807

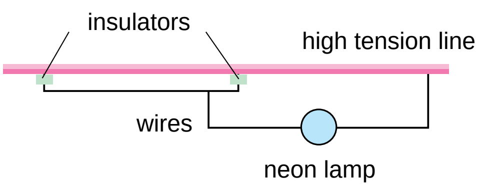

Figure 169 Small neon lamps on a high voltage cable

▪ On certain high voltage cables leading across the land scape, small neon lamps shine when the current flows. How is that possible? Challenge 808

▪ When two laser beams cross at a small angle, one can form light pulses which seem to move faster than light. Does this contradict special relativity? Ref. 420

▪ When solar plasma storms are seen on the sun, astronomers first of all phone the electricity company. They know that about 24 to 48 hours later, the charged particles ejected by the storms will arrive on earth, making the magnetic field on the surface fluctuate. Since power grids often have closed loops of several thousands of kilometres, additional electric currents are induced, which can make transformers in the grid overheat and then switch off. Then other transformers have to take over the additional power, which can lead to their overheating etc. The electricity companies avoid the problems by disconnecting the various grid sections, by avoiding large loops, by reducing the supply voltage to avoid saturation of the transformers, and by disallowing load transfer from failed circuits to others. Ref. 417

▪ Can you explain to a non-physicist how amicroscope works?* Heisenberg almost missed his PhD exam because he could not. Challenge 809

▪ Is it really possible to see stars from the bottom of a deep pit or of a well during daytime, as often stated in print? Challenge 810

▪ If one describes the electric fields as a sum of components of different frequencies, its so-called Fourier components, one finds that the amplitudes are given by Ref. 418

$$\hat{\mathbf{E}}(k,t) = \frac{1}{(2\pi)^3/2} \int \mathbf{E}(x,t) e^{-i\mathbf{kx}} d^3x \qquad (374)$$

* If not, read the beautiful text by ELIZABETH M. SLATER & HENRY S. SLATER, *Light and electron microscopy*, Cambridge University Press, 1993.

and similarly for the magnetic field. It then turns out that a Lorentz invariant quantity N, describing the energy per circular frequency ω, can be defined:

$$N = \hat{\mathbf{E}}(k,t) = \frac{1}{8\pi} \int \frac{|\mathbf{E}(k,t)|^2 + |\mathbf{B}(k,t)|^2}{c|\mathbf{k}|} d^3k \tag{375}$$

Challenge 811 Can you guess what N is physically?(Hint: think about quantum theory.)

▪ Faraday discovered how to change magnetism into electricity, knowing that electricity could be transformed into magnetism. He also found how to transform electricity into light and into chemistry. He then tried to change gravitation into electricity. But he was not successful. Why? Challenge 812

▪ Take an envelope, wet it and close it. After letting it dry for a day or more, open it in the dark. At the place where the two papers are being separated from each other, the envelope glows with a blue colour. Why? Challenge 813

▪ At high altitudes above the earth, gases are completely ionized; no atom is neutral. One speaks of ionosphere, as space is full of positive ions and free electrons. Even though both charges appear in exactly the same number, a satellite moving through the ionosphere acquires a negative charge. Why?How does the charging stop? Challenge 814

▪ A capacitor of capacity C is charged with a voltage U. The stored electrostatic energy is $E = CU^2/2$. The capacitor is then detached from the power supply and branched onto an empty capacitor of the same capacity. After a while, the voltage obviously drops to $U/2$. However, the stored energy now is $C(U/2)^2$, which is half the original value. What happened? Challenge 815

▪ Perfectly spherical electromagnetic waves are not possible. Can you show this using Maxwell's equation of electromagnetism? Challenge 816

capacitor of capacity C is charged with a voltage U. The stored electrostatic energy is $E = CU^2/2$. The capacitor is then detached from the

Is lighting a discharge? – Electricity in the atmosphere

Looking carefully, the atmosphere is full of electrical effects. The most impressive electrical phenomenon we observe, the lightning, is now reasonably well understood. Inside a thunderstorm cloud, especially inside tall *cumulonimbus* clouds,* charges are separated by collision between the falling large 'graupel' ice crystals falling due to their weight and the small 'hail' ice crystallites rising due to thermal upwinds. Since the collision takes part (Ref. 421) in an electric field, charges are separated in a way similar to the mechanism in the Kelvin generator. Discharge takes place when the electric field becomes too high, taking a strange See page 355 path influenced by ions created in the air by cosmic rays. It seems that cosmic rays are at least partly responsible for the zigzag shape of lightnings.** By the way, you have a 75%

* From Latin 'cumulus,' meaning heap, and 'nimbus', meaning big cloud. The various types of clouds all have Latin names.

** There is no ball lightning even though there is a Physics Report about them. Ball lightnings are one of the favourite myths of modern pseudo-science. Actually, they would exist if we lived in a giant microwave oven. To show this, just stick a toothpick into a candle, light the toothpick, and put it into (somebody else's) microwave at maximum power. (Ref. 422) survival chance after being hit by lightning; rapid reanimation is essential to help somebody to recover after a hit.

As a note, everybody knows how to measure the distance of a lightning by counting the seconds between the lightning and the thunder and multiplying by the speed of sound, ca. 330 m/s; it is less well known that one can estimate the *length* of the lightning bolt by measuring the *duration* of the thunder, and multiplying by it the same factor.

In the nineteen nineties, more electrical details about thunderstorms became known. Airline pilots and passengers sometime see weak and coloured light emissions spreading from the top of thunderclouds. There are various types of such emissions, blue *jets* and mostly red *sprites* and *elves*, which are somehow due to electric fields between the cloud top and the ionosphere. The details are still under investigation, and the mechanisms are not yet clear.*

All these details are part of the electrical circuit around the earth. This fascinating part of geophysics would lead us too far from the aim of our mountain ascent. But every physicist should know that there is a vertical electric field of between 100 and 300 V/m on a clear day, as discovered already in 1752. (Can you guess why it is not noticeable in everyday life? And why despite its value it cannot be used to extract large amounts of energy?)The field is directed from the ionosphere downwards to the ground; in fact the earth is permanently charged negatively, and on clear weather current flows downwards through the clear atmosphere, trying to *discharge* our planet. The current of about 1 kA is spread over the whole planet; it is possible due to the ions formed by cosmic radiation. (The resistance between the ground and the ionosphere is about 200 Ω, so that the total voltage drop is about 200 kV.) At the same time, the earth is constantly *charged* by several effects, of which the most important one turns out to be the lightning. In other words, contrary to what one may think, lightnings do not discharge the ground, they actually charge it up!** Of course, lightnings do discharge the cloud to ground potential difference, but by doing so, they actually send negative charge down to the earth. Challenge 817 Challenge 818

The electric field is an important quantity. When helicopters save people on a raft in high sea, the rope must first be earthed by hanging it in the water, otherwise the people die from electrical shock when they first touch the rope, as happened a few times in the past. Can you explain why? Challenge 820

Why are sparks and lightnings blue? This turns out to be a material property; the colour is given by the material that happens to be excited by the energy of the discharge, usually air. This excitation is due to the temperature of 30 kK inside the channel of a typical lightning stroke. For everyday sparks, the temperature is much smaller. Depending on the situation, the colour may arise from the gas between the two electrodes, such as oxygen or nitrogen, or it may due to the material evaporated from the electrodes by the discharge. For an explanation of such colours, like for the explanation of all material related colours, we need to wait for the next part of our walk.

* For images, have a look at the interesting http://sprite.gi.alaska.edu/html/sprites.htm web site.

Challenge 819 ** The earth is thus charged to about -1 MC. Can you confirm this?To learn more about atmospheric currents, you may want to have a look at the popularizing review of US work by EDGAR BERING, ARTHUR FEW & JAMES BENBROOK, *The global electric circuit*, Physics Today **51**, pp. 24–30, October 1998, or the more technical overview by EDGAR BERING, Reviews of Geophysics (supplement) **33**, p. 845, 1995.

But not only electric fields are dangerous. Also electromagnetic fields can be. In 1997, with beautiful calm weather, a Dutch hot air balloon approached the powerful radio transmitter in Hilversum. But after a few minutes near the antenna, the gondola suddenly detached from the balloon, killing all passengers inside.

An investigation team reconstructed the facts a few weeks later. In modern gas balloons the gondola is suspended by high quality nylon ropes. To avoid damage by lightning and in order to avoid electrostatic charging problems all these nylon ropes contain thin metal wires which form a large equipotential surface around the whole balloon. Unfortunately, in front of the radio transmitter these thin metal wires absorbed the radio energy from the transmitter, became red hot, and melted the nylon wires. It was the first time that this was ever observed.

Electrical nerves

In 1789 the Italian medical doctor Luigi Galvani (1737–1798) discovered that electrical current makes muscles contract. Subsequent investigations confirmed that nerves make use of electrical signals. The details were clarified only in the 20th century. Nerve signals propagate by through the motion of ions in the cell membrane making up the nerve. The resulting signal speed is between 0.5 m/s and 120 m/s, depending on the nerve type.

How to prove you're holy

Light reflection and refraction are responsible for many effects. The originally indian symbol of holiness, now used throughout most of the world, is the *aureole*, also called *halo* or *Heiligenschein*, a ring of light surrounding the head. You can easily observe it around your own head. It is sufficient to get up early one morning and to look into the wet grass while turning your back to the sun. You will see an aureole around your shadow.

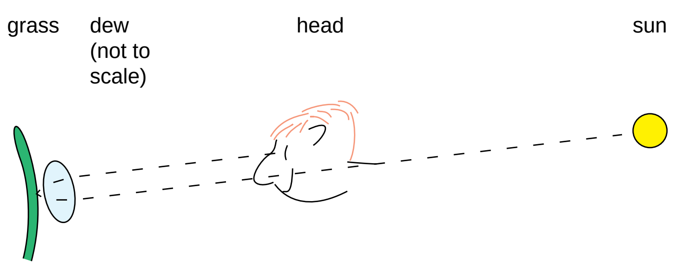

Figure 170 The path of light for dew on grass responsible for the aureole

The effect is due to the morning dew on the grass, which reflects back the light mainly into the direction of the light source, as shown in the figure. The fun part is that if one does this in a group, one sees the aureole only around one's *own* head. Ref. 423

Retroreflective paint works in the same way; it contains tiny glass spheres which play the role of the dew. A large surface of retroreflective paint, a traffic sign for example, can also show one's halo, if the light source is sufficiently far away. Also the so-called 'glow' of the eyes of cats at night is due to the same effect; it is visible only if one looks at the cat with a light source in one's back. By the way, does a cat-eye work like a cat's eye? Ref. 424 Challenge 821

Do we see what exists?

Sometimes we see *less* than there is. Close the left eye, look at the white spot in Figure 171, approach the page slowly to your eye, and pay attention to the middle lines. At a distance of about 15 to 20 cm the middle line will seem uninterrupted. Why? Challenge 822

Figure 171 A limitation of the eye

On the other hand, sometimes we see *more* than there is, as the next two figures show.

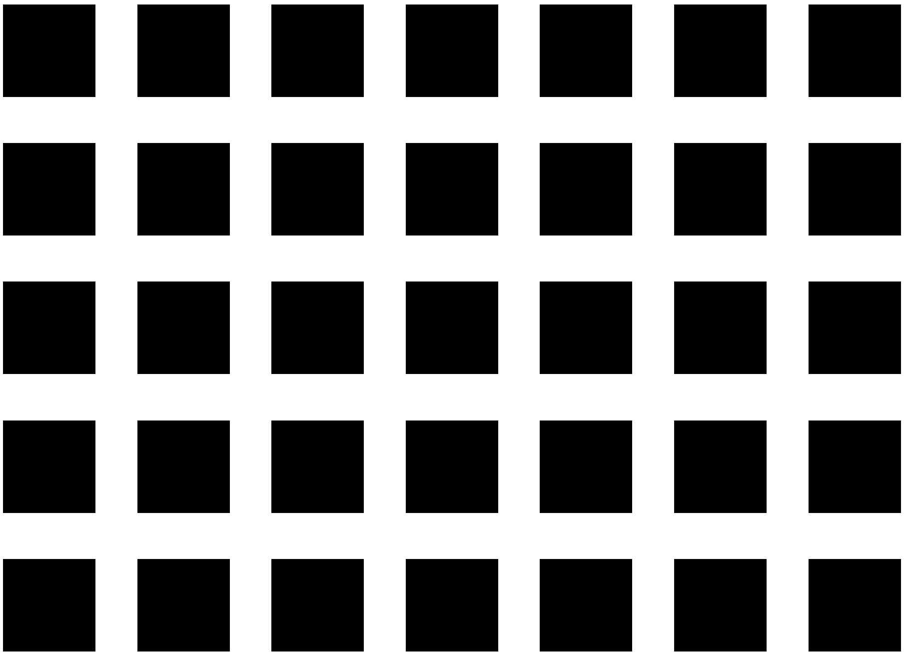

Figure 172 What is the shade of the crossings?

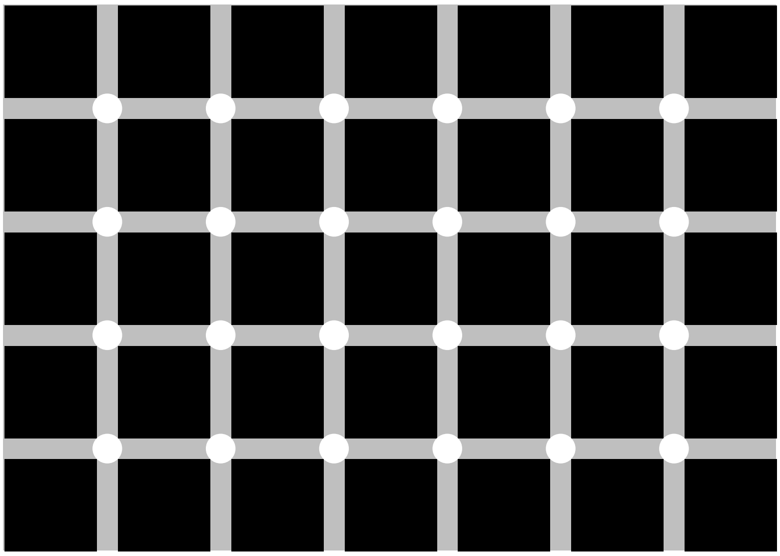

Figure 173 Do you see white, grey, or black dots?

Our eyes also sees things *differently:* the retina sees an *inverted* image of the world. There is a simple method to show this, due to Helmholtz.* You only need a needle and a piece of paper, e.g. this page of text. Use the needle to make two holes inside the two letters 'oo'. Then keep the page as near to your eye as possible, look through the two holes towards the wall, keeping the needle vertical, a few centimetres behind the paper. You will see two images of the needle. If you now cover the *left* hole with your finger, the *right* needle will disappear, and vice versa. This shows that the image inside the eye, on the retina, is inverted.

Challenge 823 Are you able to complete the proof?

We thus have to be careful when maintaining that seeing means observing. Examples such as these should make one ponder whether there could be other limitations of our senses which are less evident. And our walk will indeed uncover quite a few more.

How does one make pictures of the inside of the eye?

The most beautiful pictures so far of a *living* human retina, such as that of Figure 175, were made by the group of David Williams at the University at Rochester in New York. They used adaptive optics, a technique which changes the shape of the imaging lens in order to compensate for the shape variations of the lens in the human eye. Ref. 425

* See Hermann von Helmholtz, *Handbuch der physiologischen Optik*, 1867. This famous classic is available in English as *Handbook of physiological optics*, Dover, 1962. The Prussian physician, physicist, and science politician born as Hermann Helmholtz (1821, Potsdam–1894) was famous for his works on optics, on acoustics, electrodynamics, thermodynamics, epistemology, and geometry. He founded several physics institutions across Germany. He was one of the first to propagate the idea of conservation of energy. His other important book, *Die Lehre von den Tonempfindungen*, published in 1863, describes the basis of acoustics, and like the handbook, is still worth to be read.

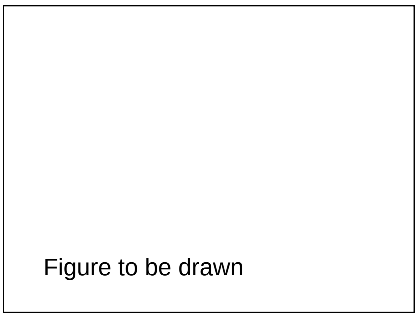

Figure 174 Eyes see inverted images

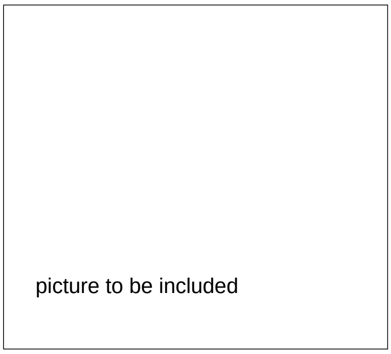

Figure 175 A high quality photograph of a live human retina

The eyes see colour by averaging the intensity arriving at the red, blue and green sensitive cones. This explains the possibility, mentioned above, to get the same impression of colour, e.g. yellow, by a pure yellow laser beam, or by the mixture of red and green light. See page 386

But if the light is focussed onto one cone only, the eye makes mistakes. If, using this adaptive optics, a red laser beam is focussed such that it hits a green cone only, a strange thing happens: even though the light is *red*, the eye sees a *green* colour!

By the way, Figure 175 is quite astounding. In the human eye, the blood vessels are located in front of the light sensitive cones. Why don't they appear in the picture? And why don't they disturb us in everyday life? Challenge 824

Amongst mammals, only primates can see *colours*. Bulls for example, don't; they cannot distinguish red from blue. On the other hand, the best colour seers overall are the birds. They have receptors for red, blue, green, UV, and depending on the bird, for up to three more sets of colours. A number of birds also have a much better eye resolution than humans.

Does gravity make charges radiate?

We learned in the section on general relativity that gravitation has the same effects as acceleration. This means that a charge kept fixed at a certain height is equivalent to a charge accelerated by $9.8\,\mathrm{m/s^2}$, which would imply that it radiates, since all accelerated charges radiate. However, the world around us is full of charges at fixed heights, and there is no such radiation. How is this possible?

The question has been a pet topic for many years. It turns out that the answer depends on whether the observer detecting the radiation is also in free fall or not, and on the precise instant this started to be the case. Ref. 426

– CS – to be filled in – CS –

How does one make holograms and other 3-d images?

Our sense of sight gives us the impression of depth mainly due to three effects. First of all, the two eyes see different images. Secondly, the images formed in the eyes are position dependent. Thirdly, our eye needs to focus differently for different distances.

The third effect is never used, as it is two weak. The first effect is used in stereo photography and in virtual reality systems, by sending two different images to the eyes. Alternatively, certain post cards and computer screens are covered by thin cylindrical lenses which allow to send two different images to the two eyes, thus generating the same effect.

But obviously the most spectacular effect is obtained whenever position dependent images can be created. Virtual reality systems mimic this effect by attaching a sensor to the head, and creating computer-generated images which depend on this position. Still, they pale when compared to the impression produced by holograms.

A *hologram* is a stored set of position dependent pictures of an objects. It is produced by storing amplitude *and phase* of the light emitted by an object. This possible if the object is illuminated by a *coherent* light source such as a laser. If the photographic film , after development, is then illuminated by a coherent or at least point-like source, one can see a full threedimensional image, often floating in free space.

Is it possible to make *moving* holograms? Yes; however, the technical set-ups are extremely expensive. By the way, can you give a simple way to distinguish a moving hologram from a real body, if you ever met one?In any case, there is no way that holograms of people, similar to ghosts, can walk around and frighten real people.

Challenge 825

Research questions

The classical description of electrodynamics is coherent and complete; nevertheless there are still many subjects of research. Here are a few.

The origin of magnetic field of the earth, the other planets, the sun, and even of the galaxy is a fascinating topic. The way that the convection of fluids inside the planets generates magnetic fields, an intrinsically three dimensional problem, the influence of turbulence, of nonlinearities, of chaos etc. makes it a surprisingly complex question.

The details of the generation of the magnetic field of the earth, usually called the *geodynamo*, began to appear only in the second half of the twentieth century, when the knowledge of the earth's interior reached a sufficient level. The earth's interior is divided into the mantle – the first 2900 km from the surface – and the core. The core is made if a liquid outer core, 2300 km thick, and a solid inner core of 1215 km radius. It seems that the liquid and electrically conducting outer core acts as a dynamo which keeps the magnetic field going. The magnetic energy comes from the kinetic energy of the outer core, which rotates with respect to the earth's surface; the fluid can act as a dynamo because, apart from rotating, it also *convects* from deep inside the earth to more shallow depths, driven by the temperature gradients between the hot inner core and the cooler mantle. Huge electric currents flow in complex ways through these liquid layers, due to friction, and create the magnetic field. Understanding why this field switches orientation at irregular intervals of between a few tens of thousands and a few million years, is one of the central questions. The answers are difficult; experiments are not possible, 150 years of measurements is a short time when

Ref. 427

compared to the last transition, about 700 000 years ago, and computer simulations are extremely involved. Since the field measurements started, the dipole moment of the magnetic field has steadily diminished, and the quadrupole moment has steadily increased. Maybe we are heading towards a surprise. By the way, the study of *galactic* magnetic fields is even more complex, and still at its beginning.

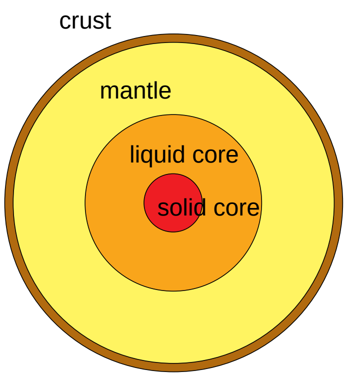

Figure 176 The structure of our planet

Another puzzle results from the equivalence of mass and energy. It is known from experiments that the size d of electrons is surely smaller than 10^{-22} m. That means that the electric field surrounding it has an energy content E given by at least Ref. 428

Challenge 826

$$E_{\text{nergy}} = \frac{1}{2}\varepsilon_o \int E^2_{\text{lectric field}}\, dV = \frac{1}{2}\varepsilon_o \int_d^\infty \left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\right)^2 4\pi r^2 dr = \frac{q^2}{8\pi\varepsilon_o}\frac{1}{d} > 1.2\,\mu\text{J} \quad . \tag{376}$$

On the other hand, the *mass* of an electron, usually given as $511\ \text{keV}/\text{c}^2$, corresponds to an energy of only 82 fJ, ten million times *less* than the value just calculated. In other words, classical electrodynamics has considerable difficulty describing electrons. In fact, a consistent description of charged point particles within classical electrodynamics is still not completely achieved. This pretty topic receives only a rare – but then often passionate – interest nowadays, because the puzzle is solved in a different way in the upcoming, second part of our mountain ascent. See page ??

Even though the golden days of materials science are over, the various electromagnetic properties of matter and their applications in devices do not seem to be completely explored yet. About once a year a new effect is discovered which merits to be included in the list of electromagnetic matter properties of Table 39. Among others, some newer semiconductor technologies will still have an impact on electronics, such as the recent introduction of low cost light detecting integrated circuits built in CMOS (complementary metal oxide silicon) technology. See page 407

Levitation

We have seen that it is possible to move certain objects without touching them, using a magnetic or an electric field, or of course, using gravity. One naturally asks if it is also possible, without touching an object, to keep it fixed, floating in mid air? Does this type of rest exist?

It turns out that there are several methods to levitate objects. They are commonly divided into two groups: those which consume energy, and those who do not. Among the methods consuming energy one has the floating of objects on a jet of air or of water, the floating of objects through sound waves, e.g. on top of a siren, or through a laser beam coming from below, and the floating of conducting material, even of liquids, in strong radiofrequency fields. The levitation of liquids or solids by strong ultrasound waves is presently becoming popular in laboratories. These methods give *stationary* levitation. Another group of energy consuming methods sense the way a body is falling and kick it up again in the right way via Ref. 429 Ref. 430

a feedback loop; these methods are *non-stationary*, and usually use magnetic fields to keep the objects from falling. The magnetic train being built in Shanghai by a German consortium
Ref. 431 is levitated this way. The whole train, including the passengers, is levitated and then moved forward with electromagnets. It is thus possible, using magnets, to levitate many tens of tons of material.

For levitation methods which do *not* consume energy – all such methods are necessarily stationary – a well-known limitation can be found studying Coulomb's 'law' of electrostatics: no static, i.e. time-independent arrangement of electric fields can levitate a *charged* object in free space or in air. The same result is valid for gravitational fields and *massive* objects;* in other words, one cannot produce a local minimum of potential energy in the middle of a box using electric or gravitational fields. This impossibility is called *Earn-*
Ref. 432 *shaw's theorem*. Speaking mathematically, the solutions of the Laplace equation $\Delta\varphi = 0$, the so-called *harmonic functions*, have minima or maxima only at the border, and never inside the domain of definition. (You proved this yourself on page 89.) The theorem can also be proved by noting that given a potential minimum in free space, Gauss' 'law' for a sphere around that minimum requires that a source of the field be present inside, which is in contradiction with the original assumption.

One can see easily that it is also impossible to use electric fields to levitate an electrically *neutral* body in air: the potential energy U of such a body, with volume V and dielectric constant ε, in an environment of dielectric constant ε_o, is given by

$$\frac{U}{V} = -\frac{1}{2}(\varepsilon - \varepsilon_o)E^2 \quad . \tag{377}$$

Challenge 827 Since the electric field E never has a maximum in the absence of space charge,and since for all materials $\varepsilon > \varepsilon_o$, there cannot be a minimum of potential energy in free space for a neutral body.**

In summary, using static electric or static gravitational fields it is impossible to keep an object from falling; neither quantum mechanics, which incorporates phenomena such as antimatter, nor general relativity, including phenomena such as black holes, change this basic result.

For static *magnetic* fields, the argument is analogous to electrical fields: the potential energy U of a magnetizable body of volume V and permeability μ in a medium with perme-
Challenge 829 ability μ_o containing no current is given by

$$\frac{U}{V} = -\frac{1}{2}\left(\frac{1}{\mu} - \frac{1}{\mu_o}\right)B^2 \tag{378}$$

and due to the inequality $\Delta B^2 \geqslant 0$, isolated maxima of a static magnetic field are not possible, only isolated minima. Therefore, it is impossible to levitate paramagnetic ($\mu > \mu_o$)

* To the disappointment of many science-fiction addicts, this would also be true in case that negative mass would exist, as happens for charge. See also page 62. And even though gravity is not really due to a field, the result still holds in general.

Ref. 433 ** It is possible, however, to 'levitate' gas bubbles in liquids – 'trap' them to prevent them from rising would be a better expression – because in such a case the dielectric constant of the environment is higher than that of
Challenge 828 the gas. Can you find a liquid-gas combination where bubbles fall instead of rising?

or ferromagnetic ($\mu \gg \mu_0$) materials such as steel, including bar magnets, which are all attracted, and not repelled to magnetic field maxima. Challenge 830

There are thus two ways to get magnetic levitation: levitating a diamagnet or using a time dependent field. Diamagnetic materials ($\mu < \mu_0$) can be levitated by static magnetic fields because they are attracted to magnetic field minima; the best-known example is the levitation of superconductors, which are, at least those of type I, perfects diamagnets ($\mu = 0$). Strong forces can be generated, and this method is also being tested for the levitation of passenger trains in Japan. In some cases, superconductors can even be *suspended* in mid-air, below a magnet. Single atoms with a magnetic moment are also diamagnets; they are routinely levitated this way and have also been photographed in this state. Ref. 434 Ref. 431 Ref. 435

Also single neutrons, which have a magnetic dipole moment, have been kept in magnetic bottles in this way, until they decay. Recently, people have levitated pieces of wood, of plastic, strawberries, water droplets, liquid helium droplets as large as 2 cm, grasshoppers, fish, and frogs (all alive an without any harm) in this way. They are, like humans, all made of diamagnetic material. Humans themselves have not yet been levitated, but the feat is being planned and worked on. Ref. 436

Diamagnets levitate if $\nabla B^2 > 2\mu_0 \rho g/\chi$, where ρ is the mass density of the object and $\chi = 1 - \mu/\mu_0$ its magnetic susceptibility.Since χ is typically about 10^{-5} and ρ of order $1000\ \mathrm{kg/m^3}$, one needs field gradients of about $1000\ \mathrm{T^2/m}$. In other words, levitation requires fields changes of 10 T over 10 cm, nowadays common for high field laboratory magnets. Challenge 831

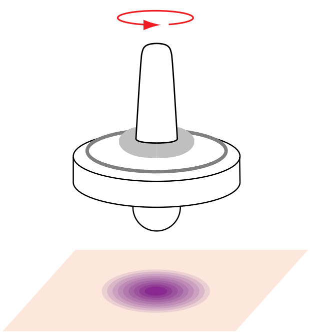

Figure 177 Floating 'magic' nowadays available in toy shops

Finally, *time dependent* electrical or magnetic fields, e.g. periodic fields, can lead to levitation in many different ways without any consumption of energy. This is one of the methods used in the magnetic bearings of turbomolecular vacuum pumps. Also single charged particles, such as ions and electrons, are now regularly levitated with Paul traps and Penning traps. The mechanical analogy is shown in Figure 178. Ref. 429 Ref. 429

Figure 177 shows a toy allowing to let one personally levitate a spinning top in mid air above a ring magnet, a quite impressive demonstration of levitation for anybody looking at it. It is not hard building such a device oneself. Ref. 437 Ref. 438

Even free electrons can be levitated, letting them float above the surface of fluid helium. In the most recent twist of the science of levitation, in 1995 Stephen Haley predicted that the suspension height of small magnetic particles above a superconducting ring should be quantized. However, the prediction has not been checked by experiment yet. Ref. 439

For the sake of completeness we mention that the nuclear forces cannot be used for levitation in everyday life, as their range is limited to a few femtometres. However, we will see later that the surface matter of the sun is prevented from falling into the centre by these interactions; one could thus say that it is indeed levitated by nuclear interactions. See page 644

Matter, levitation and electricity

Levitation used by magicians mostly falls into another class. When David Copperfield, a magician performing for the MTV generation at the end of the twentieth century, ‘flies’ during his performances, he does so by being suspended on thin fishing lines kept invisible by Ref. 440 clever lighting arrangements. In fact, if one wants to be precise, one should count fishing lines as well as any table as levitation devices. Contrary to impression, a hanging or lying object is not really in contact with the suspension, if one looks at the critical points with a microscope. More about this in the next part of our walk.

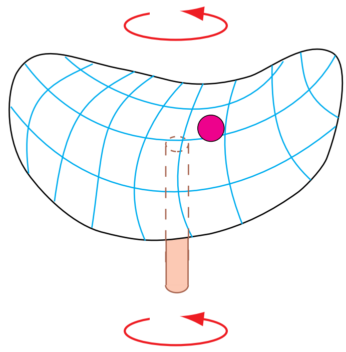

Figure 178 Trapping a metal sphere using a variable speed drill and a plastic saddle

But if this is the case, why don’t we fall through a table or through the floor? We started the study of mechanics by stating as key property of matter its *solidity*, i.e. the impossibility to have more than one body at the same place at the same time. But what is the origin of solidity? Again, we will be able to an- See page 590 swer the question only in the part on quantum mechanics, but we can collect the first clues already at this point.

Many experiments show that matter is constituted of charged particles; indeed, matter can be moved and influenced by electromagnetic fields in many ways. Over the years, material scientists have produced a long list of such effects, all of which are based on the existence of Challenge 832 charged constituents.* Can you find or imagine a new one?For example, can electric charge change the colour of objects?

Table 39 Selected matter properties related to electromagnetism, showing among others the role it plays in the constitution of matter; at the same time a short overview of atomic, solid state, fluid and business physics

Name of property	example	definition
thermal radiation or heat radiation or incandescence	every object	temperature dependent radiation emitted by any macroscopic amount of matter
Interactions with charges and currents		
electrification	separating metals from insulators	spontaneous charging
triboelectricity	glass rubbed on cat fur	charging through rubbing
barometer light	mercury slipping along glass	gas discharge due to triboelectricity Ref. 441
insulation	air	no current flow below critical voltage drop
semiconductivity	diamond, silicon or gallium arsenide	current flows only when material is impure (‘doped’)

* Detailed descriptions of many of these effects can be found in the excellent overview edited by MANFRED VON ARDENNE, GERHARD MUSIOL & SIEGFRIED REBALL, *Effekte der Physik und ihre Anwendungen*, Harri Deutsch, 1997.

Name of property	example	definition
conductivity	copper, metals	current flows easily
superconductivity	niobium	current flows indefinitely
ionisation	fire flames	current flows easily
localization (weak, Anderson)	disordered solids	
resistivity, Joule effect	graphite	heating due to current flow
thermoelectric effects: Peltier effect, Seebeck effect, Thomson effect	ZnSb, PbTe, PbSe, BiSeTe, etc.	cooling due to current flow, current flow due to temperature difference, or due to temperature gradients
acoustoelectric effect	CdS	sound generation by currents, and vice versa
magnetoresistance	iron, metal multilayers	resistance changes with applied magnetic field Ref. 442
recombination	fire alarms	charge carriers combine to neutral atoms or molecules
annihilation	positron tomography	particle and antiparticle, e.g. electron and positron, disappear into photons
Penning effect	Ne, Ar	ionisation through collision with metastable atoms
Richardson effect, thermal emission	BaO_2, W, Mo, used in tv and electron microscopes	emission of electrons from hot metals
skin effect	Cu	high current density on exterior of wire
pinch effect	InSb, plasmas	high current density on interior of wire
Josephson effect	Nb-Oxide-Nb	tunnel current flows through insulator between two superconductors
Sasaki-Shibuya effect	n-Ge, n-Si	anisotropy of conductivity due to applied electric field
switchable magnetism	InAs:Mn	voltage switchable magnetization Ref. 443
Interactions with magnetic fields		
Hall effect	silicon; used for magnetic field measurements	voltage perpendicular to current flow in applied magnetic field
Zeeman effect	Cd	change of emission frequency with magnetic field
Paschen-Back effect	atomic gases	change of emission frequency in strong magnetic fields
ferromagnetism	Fe, Ni, Co, Gd	spontaneous magnetization; material strongly attracted by magnetic fields
paramagnetism	iron	induced magnetization parallel to applied field; attracted by magnetic fields
diamagnetism	water	induced magnetization opposite to applied field; repelled by magnetic fields
magnetostriction	CeB_6, $CePd_2Al_3$	change of shape or volume by applied magnetic field
magnetoelastic effect	Fe, Ni	change of magnetization by tension or pressure

Name of property	example	definition
acoustomagnetic effect	metal alloys, anti-theft etiquettes	excitation of mechanical oscillations through magnetic field
spin valve effect	metal multilayers	electrical resistance depends on spin direction of electrons with respect to applied magnetic field
magnetooptical activity or Faraday effect or Faraday rotation	flint glass	polarization angle is rotated with magnetic field; different refraction index for right and left circularly polarized light, as in magnetooptic (MO) recording
magnetic circular dichroism	gases	different absorption for right and left circularly polarized light; essentially the same as the previous one
photoelectromagnetic effect	InSb	current flow due to light irradiation of semiconductor in a magnetic field
Voigt effect	vapours	birefringence induced by applied magnetic field
Cotton-Mouton effect	liquids	birefringence induced by applied magnetic field
Hanle effect	Hg	change of polarization of fluorescence with magnetic field
Shubnikov-de Haas effect	Bi	periodic change of resistance with applied magnetic field
thermomagnetic effects: Ettinghausen effect, Righi-Leduc effect, Nernst effect, magneto-Seebeck effect	BiSb alloys	relation of temperature, applied fields, and electric current
Ettinghausen-Nernst effect	Bi	appearance of electric field in materials with temperature gradients in magnetic fields
photonic Hall effect	CeF_3	transverse light intensity depends on the applied magnetic field Ref. 444
magnetocaloric effect	gadolinium, GdSiGe alloys	material cools when magnetic field is switched off Ref. 445
cyclotron resonance	semiconductors, metals	selective absorption of radio waves in magnetic fields
magnetoacoustic effect	semiconductors, metals	selective absorption of sound waves in magnetic fields
magnetic resonance	most materials, used for imaging in medicine for structure determination of molecules	selective absorption of radio waves in magnetic fields
magnetorheologic effect	liquids, used in advanced car suspensions	change of viscosity with applied magnetic fields

Name of property	example	definition
Meissner effect	type 1 superconductors, used for levitation	expulsion of magnetic field from superconductors
Interactions with electric fields		
polarizability	all matter	polarization changes with applied electric field
ionization, field emission, Schottky effect	all matter, tv	charges are extracted at high fields
paraelectricity	$BaTiO_3$	applied field leads to polarization in same direction
dielectricity	water	in opposite direction
ferroelectricity	$BaTiO_3$	spontaneous polarization below critical temperature
piezoelectricity	like the quartz lighter used in the kitchen	polarization appears with tension, stress, or pressure
pyroelectricity	$CsNO_3$, tourmaline, crystals with polar axes; used for infrared detection	change of temperature produces charge separation
electroosmosis or electrokinetic effect	many ionic liquids	liquid moves under applied electric field Ref. 446
electrowetting	salt solutions on gold	wetting of surface depends on applied voltage
electrolytic activity	sulfuric acid	charge transport through liquid
liquid crystal effect	watch displays	molecules turn with applied electric field
electrooptical activity: Kerr effect, Pockels effect	liquids (e.g. oil), crystalline solids	material in electric field rotates light polarisation, i.e. produces birefringence
Freederichsz effect, Schadt-Helfrichs effect	nematic liquid crystals	electrically induced birefringence
Stark effect	hydrogen, mercury	colour change of emitted light in electric field
field ionisation	helium near tungsten tips in field ion microscope	ionisation of gas atoms in strong electric fields
Zener effect	Si	energy-free transfer of electrons into conduction band at high fields
field evaporation	W	evaporation under strong applied electric fields
Interactions with light		
absorption	coal, graphite	transformation of light into heat or other energy forms (which ones?) Challenge 833
blackness	coal, graphite	complete absorption in visible range
colour, metallic shine	ruby	absorption depending on light frequency
photostriction	PbLaZrTi	light induced piezoelectricity
photography	AgBr, AgI	light precipitates metallic silver
photoelectricity, photoeffect	Cs	current flows into vacuum due to light irradiation

Name of property	example	definition
internal photoelectric effect	Si p-n junctions, solar cells	voltage generation and current flow due to light irradiation
photon drag effect	p-Ge	current induced by photon momentum
emissivity	every body	ability to emit light
transparency	glass, quartz, diamond	low reflection, low absorption, low scattering
reflectivity	metals	light bounces on surface
polarization	pulled polymer sheets	light transmission depending on polarization angle
optical activity	sugar dissolved in water, quartz	rotation of polarization
birefringence	feldspat,cornea	refraction index depends on polarization direction, light beams are split into two beams
dichroism	feldspat	absorption depends on polarisation
optically induced anisotropy, Weigert effect	AgCl	optically induced birefringence and dichroism
second harmonic generation	$LiNbO_3$, KPO_4	light partially transformed to double frequency
luminescence: general term for the opposite of incandescence	GaAs, television	cold light emission
fluorescence	CaF_2, X ray production, light tubes, cathode ray tubes	light emission during and after light absorption or other energy input
phosphorescence	$TbCl_3$	light emission due to light, electrical or chemical energy input, continuing *long after* stimulation
electroluminescence	ZnS	emission of light due to alternating electrical field
also photo-, chemo-, tribo-, bio-, thermoluminescence		
thermoluminescence	quartz, feldspat	light emission during heating, used e.g. for archaeological dating of pottery Ref. 447
Bremsstrahlung	X ray generation	radiation emission through fast deceleration of electrons
Compton effect	momentum measurements	change of wavelength of light, esp. X rays and gamma radiation, colliding with matter
Cerenkov effect	water, polymer particle detectors	light emission in a medium due to particles, e.g. emitted by radioactive processes, moving faster than the speed of light in that medium
transition radiation	any material	light emission due to fast particles moving from one medium to a second with different refractive index
electrochromicity	wolframates	colour change with applied electric field
scattering	gases, liquids	light changes direction
Mie scattering	dust in gases	light changes direction
Raleigh scattering	sky	light changes direction, sky is blue

Name of property	example	definition
Raman effect or Smekal-Raman effect	molecular gases	scattered light changes frequency
laser activity, superradiation	beer, ruby, He-Ne	emission of stimulated radiation
sonoluminescence	air in water	light emission during cavitation
gravitoluminescence	fake; it does not exist; why? Challenge 834	
switchable mirror	LaH	voltage controlled change from reflection to transparency Ref. 448
radiometer effect	bi-coloured windmills	mill turn due to irradiation (see page 385)
luminous pressure	idem	opposite of the preceding one
solar sail effect	future satellites	motion due to solar wind
acoustooptic effect	$LiNbO_3$	diffraction of light by sound in transparent materials
photorefractive materials	$LiNbO_3$, GaAs, InP	light irradiation changes refractive index
Auger effect	Auger electron spectroscopy	electron emission due to atomic reorganisation after ionisation by X rays
Bragg reflection	crystal structure determination	X ray diffraction by atomic planes
Mößbauer effect	Fe, used for spectroscopy	recoil-free resonant absorption of gamma radiation
pair creation	Pb	transformation of a photon in a charged particle-antiparticle pair
photoconductivity	Se, CdS	change of resistivity with light irradiation
optoacoustic affect, photoacoustic effect	gases, solids	creation of sound due to absorption of pulsed light
optogalvanic effect	plasmas	change of discharge current due to light irradiation
optical nonlinear effects: parametric amplification, frequency mixing, saturable absorption, n-th harmonic generation, optical Kerr effect, etc.		
phase conjugated mirror activity	gases	reflection of light with opposite phase
solidity, impenetrability	floors, columns, ropes, buckets	at most one object per place at a given time
Interactions with vacuum		
Casimir effect	metals	attraction of uncharged, conducting bodies

All matter properties in the list can be influenced by electric or magnetic fields or directly depend on them. This shows that the nature of all these material properties is electromagnetic. In other words, charges and their interactions are an essential and fundamental part of the structure of objects. The table shows so many different electromagnetic properties that the motion of charges inside each material must be complex indeed. Most effects are the topic of solid state physics,* fluid and plasma physics.

* Probably the best and surely the most entertaining introductory English language book on the topic is the one by Neil Ashcroft & David Mermin, *Solid state physics*, Holt Rinehart & Winston, 1976.

Solid state physics is by far the most important part of physics, when measured by the impact it had on society. Almost all effects have applications in technical products, and give work to many people. Can you name a product or business application for any randomly
Challenge 835 choseneffect from the table?

In our mountain ascent we however, we look only at one example from the above list: thermal radiation, the emission of light by hot bodies.

Earnshaw's theorem about the impossibility of a stable equilibrium for charged particles at rest implies that the charges inside matter must be *moving*. For any charged particle in motion, Maxwell's equations for the electromagnetic field show that it radiates energy by emitting electromagnetic waves. In short, classical mechanics thus predicts that matter must radiate electromagnetic energy.

Interestingly, everybody knows from experience that this is indeed the case. Hot bodies light up depending on their temperature; the fact that light bulbs work thus proves that metals are made of charged particles. *Incandescence*, as it is called, requires charges. Actually, *every* body emits radiation, even at room temperature. This radiation is called *thermal radiation*; at room temperature it lies in the infrared. Its intensity is rather weak in everyday
Ref. 449 life; it is given by the general expression

$$I(T) = fT^4\,\frac{2\pi^5 k^4}{15c^2h^3} \quad \text{or} \quad I(T) = f\sigma T^4 \quad \text{with} \quad \sigma = 56.7\,\mathrm{nW/K^4m^2} \tag{379}$$

where f is a material, shape, and temperature dependent factor, with a value between zero and one, and called the *emissivity*. A body whose emissivity is given by the ideal case $f = 1$ is called a *black body*, because at room temperature such bodies also have an ideal
Challenge 836 absorption coefficient and thus appear black. (Can you see why?)The heat radiation they emit is called *black body radiation*.

Ref. 450 By the way, which object radiates more energy: a human body or an average piece of the
Challenge 837 sun of the same mass?Try to guess first.

Why can we see each other?

This use of the term 'black' is rather strange, since it turns out that most bodies at temperatures at which they are red hot or even hotter are good approximations of black bodies! For example, the tungsten in incandescent light bulbs, at around 2000 K, emits almost pure black body radiation; however, the glass then absorbs much of the ultraviolet and infrared components. Black bodies are also used to define the colour *white*. What we commonly call pure white is the colour emitted by a black body of 6500 K, namely the sun. This definition
Ref. 451 is used throughout the world, e.g. by the Commission Internationale d'Eclairage. Hotter black bodies are bluish, colder ones are yellow, orange or red.* The stars in the sky are classified in this way, as summarized on page 127.

Let us have a quick summary of black body radiation. Black body radiation has two important properties; first, the emitted power increases with the fourth power of the temperature. With this power relation alone you can check the just mentioned temperature of the

* Most bodies are not black, because colour is not only determined by emission, but also by absorption of light.

sun simply by comparing the size of the sun with the width of your thumb when the arm is stretched away from the face. Are you able to do this?(Hint: use the excellent approximation that the earth's temperature of about 300 K is due to the sun's irradiation.)* Challenge 838

The precise expression for the emitted energy density u per frequency ν can be deduced from the radiation law for black bodies discovered by Max Planck in 1899:

$$u(\nu,T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1} \quad . \tag{380}$$

He made this important discovery, which we will discuss in more detail in the second part of our mountain ascent, simply by comparing this curve with experiment.** The new constant h, *Planck's quantum of action* or *Planck's constant*, turns out to have the value $6.6\cdot 10^{-34}$ Js, and is central to all quantum theory, as we will see. The other constant Planck introduced, the Boltzmann constant k, appears as a prefactor of temperature all over thermodynamics, as it acts as a conversion unit from temperature to energy. See page 500

The radiation law gives for the total emitted energy density the expression Challenge 839

$$u(T) = T^4\frac{8\pi^5k^4}{15c^3h^3} \tag{381}$$

from which equation (379) is deduced using $I = uc/4$. (Why?) Challenge 840

The second property of black body radiation is the value of the peak wavelength, i.e. the wavelength emitted with the highest intensity. This wavelength determines their colour; it is deduced from equation (380) to be Challenge 841

$$\lambda_{\max} = \frac{hc}{4.956\,k}\frac{1}{T} = 2.9\,\mathrm{mm\,K}/T \quad \text{but} \quad \hbar\nu_{\max} = 2.82\,kT = 3.9\cdot 10^{-23}\,\mathrm{J/K}\,T \quad . \tag{382}$$

Either of these expressions is called *Wien's colour displacement* after its discoverer.*** For 37 °C, human body temperature, it gives a peak wavelength of 9.3 μm, which is thus the colour of the bulk of the radiation emitted by every human being. (Note that the peak wavelength does not correspond to the peak frequency. Why?)On the other hand, following the telecommunication laws of many countries, any radiation emitter needs a licence to operate; as a consequence in Germany only dead people are legal, and only if their bodies are at absolute zero temperature. Challenge 842

Note that a black body (also a star) can be blue, white, yellow, orange or red. It is never green. Can you explain why? Challenge 843

Ref. 452 * The actual average temperature of the earth is 14.0 °C.

** Max Planck (1858–1947), professor of physics in Berlin, was a central figure in thermostatics. He discovered and named *Boltzmann's constant k* and the *quantum of action h*, often called Planck's constant. His introduction of the quantum hypothesis was the birth date of quantum theory. He also made the works of Einstein known in the physical community, and later organized a job for him in Berlin. He received the Nobel prize for physics in 1918. He was an important figure in the German scientific establishment; he also was one of the very few who had the courage to tell Hitler *face to face* that it was a bad idea to fire Jewish professors. Famously modest, with many tragedies in his personal life, he was esteemed by everybody who knew him.

*** Wilhelm Wien (1864, Gaffken–1824, München), east Prussian physicist, received the Nobel prize for physics in 1911 for the discovery of this relation.

Above, we predicted that any material made of charges emits radiation. Are you able to find a simple argument showing whether heat radiation is or is not this classically predictedradiation?
Challenge 844

But let us come back to the question in the section title. The existence of thermal radiation implies that any hot body will cool, even if it is left in the most insulating medium there is, namely in vacuum. More precisely, if the vacuum is surrounded by a wall, a body in the vacuum will gradually approach the same temperature as the wall.

Interestingly, when the temperature of the wall and of the body inside have become the same, something strange happens. The effect is difficult to check at home, but impressive photographs exist in the literature.
Ref. 453

An arrangement in which the walls and the objects inside are at the same temperature is called an *oven*. It turns out that one *cannot see* objects in an oven using the light coming from thermal radiation. For example, if an oven and all its contents are red hot, taking a picture of the inside of the oven (without a flash!) does not reveal anything; no contrast nor brightness changes exist which allow to distinguish the objects from the walls or their surroundings. Can you explain the finding?
Challenge 845

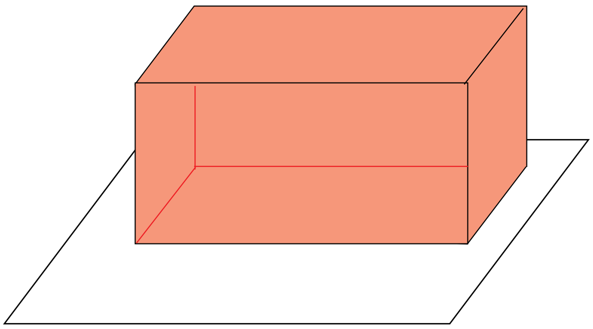

Figure 179 Hot bodies in a hot oven

In short, we are able to see each other only because the light sources we use are at a *different* temperature than ourselves. We can see each other only because we do *not* live in thermal equilibrium with our environment.

Could electrodynamics be different?

Any interaction like Coulomb's rule (329) which acts, for one given observer, between two particles independently of 3-velocity, must depend on 3-velocity for other inertial observers.* It turns out that such an interaction cannot be independent of the 4-velocity either. Such an interaction, even though it would indeed be 3-velocity dependent, would change the rest mass, since the 4-acceleration would not be 4-orthogonal to the 4-velocity.

The next simplest case is the one in which the acceleration is proportional to the 4-velocity. Together with the request that the interaction leaves the rest mass constant, one then recovers electrodynamics.
Ref. 454

In fact, also the requirements of gauge symmetry and of relativity symmetry make it impossible to modify electrodynamics. In short, it does not seem possible to have a behaviour different from $1/r^2$ for a classical interaction.

* This can be deduced from the special relativity in various ways, e.g. from the reasoning of page 366, or the formula in the footnote of page 209.

A small non-vanishing mass for the photon would change electrodynamics somewhat. Experiments pose tight limits on the mass value, but the inclusion of a tiny mass poses no special problems, and the corresponding Lagrangian has already been studied in the literature, just in case.

A summary of classical electrodynamics and of its limits

In general, classical electrodynamics can be summarized in a few main ideas.

– The electromagnetic field is a physical observable, as shown e.g. by compass needles;

– its sources are the (moving) charges, described by Maxwell's evolution equations, as shown e.g. by the properties of amber, lodestone, batteries, and remote controls;

– the electromagnetic field changes the motion of electrically charged objects via the Lorentz expression, as e.g. shown by electric motors;

– it behaves like a continuous quantity, a distribution of little arrows, and propagates as a wave, as shown e.g. by radios;

– it can exist and move also in empty space, as shown e.g. by the stars.

However, there is quite some fun ahead; even though this description is correct in everyday life, during the rest of our mountain ascent we will find that *each* of these ideas is in fact wrong. A simple example shows the trouble ahead.

At a temperature of zero Kelvin, when matter does not radiate thermally, one has the paradoxical situation that the charges inside matter cannot be moving, since no emitted radiation is observed, but they cannot be at rest either, due to Earnshaw's theorem. In short, the fact that matter actually exists shows that classical electrodynamics is wrong.

In fact, Table 39, giving an overview of material properties, makes the same point even See page 407
more strongly; classical electrodynamics can describe many of the effects listed, *but it cannot explain the origin of any of them*. Even though few of the effects will be studied in our walk – they are not essential for our adventure – the general concepts necessary for their description will be the topic of the second part of this mountain ascent, that on quantum theory.

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401 There are also other ways to see the green ray, for longer times, namely when a fata morgana appears at sunset. An explanation with colour photograph is contained in M. VOLLMER, *Gespiegelt in besonderen Düften ...– Oasen, Seeungeheuer und weitere Spielereien der Fata Morgana*, Physikalische Blätter **54**, pp. 903–909, 1998. Cited on page 387.

402 This famous discovery is by BRENT BERLIN & PAUL KAY, *Basic color terms: their universality and evolution*, 1969. Their ongoing *world colour survey* is eagerly awaited. Of course there also are ongoing studies to find possible exceptions; the basic structure is solid, as shown in the conference proceedings C.L. HARDIN & LUISA MAFFI, *Color categories in thought and language*, Cambridge University Press, 1997. Cited on page 388.

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405 Y.P. TERLETSKII, *Paradoxes in the theory of relativity*, Plenum Press, New York, 1968, Cited on page 390.

406 See the excellent explanation by KIRK T. MCDONALD, *Negative group velocity*, American Journal of Physics **69**, pp. 607–614, 2001. Cited on page 390.

407 The explanation with different refraction directions was published by P. VALANJU & al., Physical Review Letters **88**, p. 187401, 2002. Two examples of materials with negative diffraction indices were 'presented' (probably by conviction, not by fraud) by David Smith and his team; even more strangely, the theoretician John Pendry had predicted that such materials would allow to make perfect lenses, without any limit on resultion, and even allowing to use evanescent waves for imaging. Today one can say that these supposed consequences just underline that negative refraction indices are impossible. Cited on page 390.

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409 A summary of all evidence about the motion of the aether is given by R.S. SHANKLAND & al., *New analysis of the interferometer observations of Dayton C. Miller*, Review of Modern Physics **27**, p. 167, 1955. An older text is H. WITTE, Annalen der Physik **26**, p. 235, 1908. Cited on page 392.

410 The history of the concept of vacuum can be found in the book by E. GRANT, *Much ado about nothing*, Cambridge University Press, 1981, and in the extensive text by EDMUND T. WHITTAKER, *A history of the theories of aether and electricity*, Volume 1: *The classical theories*, Volume 2: *The modern theories*, Tomash Publishers, American Institute of Physics 1951, 1987. Cited on page 393.

The various *aether models* – gears, tubes, vortices – proposed in the 19th century were dropped for various reasons. Since many models used to explain electric and magnetic fields as motion of some entities, it was concluded that the speed of light would depend on electric or magnetic fields. One type of field was usually described by linear motion of the entities, the other by rotatory or twisting motion; both assignments are possible. As a consequence, aether must be a somewhat strange fluid which flows perfectly, but which resists rotation of volume elements, as McCullogh deduced in 1839. However, experiments show that the speed of light in vacuum does not depend on electromagnetic field intensity. Vortices were dropped because real world vortices were found out to be unstable. All models received their final blow when they failed to meet the requirements of special relativity.

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412 D. SINGLETON, *Electromagnetic angular momentum and quantum mechanics*, American Journal of Physics **66**, pp. 697–701, 1998, Cited on page 394.

413 A. YAZDANI, D.M. EIGLER & N.D. LANG, *Off-resonance conduction through atomic wires*, Science **272**, pp. 1921–1924, 28 June 1996. For aluminium, gold, lead, niobium, as well as the influence of chemical properties, see ELKE SCHEER, *The signature of chemical valence in the electric conduction through a single-atom contact*, Nature **394**, pp. 154–157, 9 July 1998. Cited on page 395.

414 See L. KOWALSKI, *A myth about capacitors in series*, The Physics Teacher **26**, pp. 286–287, 1988, and A.P. FRENCH, *Are the textbook writers wrong about capacitors?*, The Physics Teacher **31**, pp. 156–159, 1993. Cited on page 395.

415 This problem was suggested by Vladimir Surdin. Cited on page 395.

416 PHILIP COHEN, *Open wide, this won't hurt a bit*, New Scientist p. 5, 3 February 1996. Cited on page 396.

417 The Prussian explorer Alexander von Humboldt extensively checked this myth in the 19th century. He visited many mine pits, and asked countless mine workers in Mexico, Peru, and Siberia about their experiences. He also asked numerous chimney-sweeps. Neither him nor anybody else had ever seen the stars during the day. Cited on page 396.

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420 D. Mugnai, A. Ranfagni & R. Ruggieri, Physical Review Letters **84**, p. 4830, 2000. Cited on page 396.

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422 B.M. Smirnov, *Physics of ball lightning*, Physics Reports **224**, pp. 151–236, 1993. See also D. Finkelstein & J. Rubinstein, *Ball lightning*, Physical Review **135**, pp. 390–396, 1964. For more folklore on the topic, just search the world wide web. Cited on page 397.

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425 David R. Williams, *Supernormal Vision*, Science News **152**, pp. 312–313, 15 November 1997. See also http://www.cvs.rochester.edu/people/d_williams/d_williams.html as well as the photographs of the interior of living human eyes shown on http://www.cvs.rochester.edu/people/~ aroorda/ao_research.html Cited on page 401.

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449 See any book on thermostatics, such as LINDA REICHL, *A modern course in statistical physics*, Wiley, 2nd edition, 1998. Cited on page 413.

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454 WOLFGANG RINDLER, *Essential relativity – special, general, and cosmological*, revised second edition, Springer Verlag, 1977, page 247. There is also the beautiful paper by M. LE BELLAC & J.-M. LEVY-LEBLOND, *Galilean electrodynamics*, Nuovo Cimento B **14**, p. 217, 1973, which explains the possibilities one has and the problems one gets when trying to define the theory non-relativistically. Cited on page 415.

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Chapter IV Classical physics in a nutshell

The description of general relativity and classical electrodynamics concludes our walk through classical physics.* In order to see its limits, we summarize what we have learned about motion so far.

In every example of motion, we distinguish the moving and localized entity, the object, from the extended environment. For each of them we distinguish the fixed, intrinsic properties from the varying state.

Looking for all the *fixed, intrinsic* aspects of objects, we find that all sufficiently small objects or particles are described completely by their mass and their electric charge. There is no magnetic charge. In summary, mass and electric charge are the only localized intrinsic properties of classical, everyday objects. Both mass and electric charge are defined by the accelerations they produce around them. Both quantities can vary continuously, are conserved, and can be added. They are thus described by real numbers. Mass, in contrast to charge, is always positive. Both mass and charge describe the interaction of particles with the environment, i.e. with fields, and thus indirectly with other particles. Extended objects are described by continuous mass and charge distributions.

Looking for all *varying* aspects of objects, i.e. for their state, we find that we can describe them completely, at each instant of time, using only two basic aspects: the momentum and the position. Momentum and position can vary continuously in amount and orientation; observing how these aspects are described by different observers, we find that they are completely characterized by three-dimensional vectors. The set of all possible states is called the phase space. The phase space is described by continuous manifolds. The state of large objects made of more than one constituent is given by the states of all its constituent particles. These particles make up all objects and somehow interact electromagnetically.

The state of a particle depends on the observer. However, the states of the same particle found by different observers are related: the relations are called the 'laws' of motion. For example, for different times they are called *evolution equations*, for different places and orientations they are called *transformation relations*, and for different gauges they are called *gauge transformations*.

* Others prefer to include in classical physics only *special* relativity; this is a matter of personal preference.

Apart from the motion of massive objects, we observe motion of a massless entity: *radiation*. All types of radiation, such as light, radio waves, and their related forms, are travelling electromagnetic waves, and are described by same equations that describe the interaction of charged or magnetic objects. The speed of massless entities is the maximum possible speed and is the same for all observers. The *state* of radiation is described by the electromagnetic field strength. The *intrinsic properties* of radiation are the field strength, its polarization, and its coupling to matter. The motion of radiation describes the motion of images.

The *environment* is described by space and time coordinates. The three spatial and the single temporal coordinate characterize a curved space-time. Space-time turns out to be able to move as well, in form of gravity waves. Space and time are described by entities which are continuous, extended, and which allow to define distances. Their intrinsic properties are the number of dimensions, its signature, and its topology. Their state is given by the metric, which describes the local warpedness. The warpedness can change, so that it is fair to say that empty space can move like a wave.

We learned that our environment is finite in age. We learned the main lines of its history, and the fact that on large scales, the matter in the universe moves away from the surrounding matter. Finally we discovered that we do not know yet the large scale topology of our environment, nor do we know what happens at its spatial and temporal limits.

The motion of *objects* is described by several simple relations. First of all, no two objects can be at the same point at the same time. Secondly, masses move the way space-time tells them, and space moves the way masses tell it. This relation describes the motion of the stars, of thrown stones, of the tides, etc. Thirdly, mass is needed to break the conformal symmetry, and to distinguish space from time.

We learned that electromagnetism is necessary to define length and time intervals, that light travels at the maximum possible velocity, and that rest and free fall are the same, and that gravity is curved space-time. In summary, we learned that of the two naive types of object motion, namely motion due to interaction with space-time curvature and motion due to the electromagnetic field, only the latter is genuine.

We also saw that speeds in nature are bound from above by a universal constant c, and that length to mass ratios are bound from below by a universal constant $4G/c^2$.

Above all, classical physics showed us that motion, be it linear or rotational, be it that of matter, of radiation, or of space-time, is conserved. It is similar to a continuous substance: it is never destroyed, never created, and always only redistributed. Due to conservation, all motion, that of objects, of images, and of empty space, is predictable and reversible. Due to conservation of motion, time and space can be defined. We also found that classical motion is right-left symmetric. In summary, despite everyday experience, there are *no* surprises in nature.

The future of planet earth

Maybe nature shows no surprises, but still provides many adventures. On the 8th of march 2002, a 100 m sized body almost hit the earth. It passed at a distance of only 450 000 km. On impact, it would have destroyed a region of the size of Berlin. A few months earlier, a

300 m sized body missed the earth by 800 000 km; the record so far was in 1994, when the distance was only 100 000 km.*

Several other disasters can be predicted by classical physics, as shown in Table 40. Most are problems facing humanity only in a distant future. Nevertheless, all are research topics.

Table 40 Some examples of disastrous motion of possible future importance

Critical situation	time scale in years from now
▪ end of physics	ca. 50 (ca. year 2050)
▪ ozone shield reduction	ca. 100
▪ rising ocean levels due to greenhouse warming	ca. 100-1000
▪ explosion of volcano in Greenland, leading to darkening of sky	unknown
▪ several magnetic north and south poles, allowing solar storms to disturb radio and telecommunications, to interrupt electricity supplies, to increase animal mutations, and to disorient migrating animals such as wales, birds and tortoises	ca. 800
▪ our interstellar gas cloud detaches from the solar systems, changing the size of the heliosphere, and thus auroras and solar magnetic fields	ca. 3000
▪ subsequent reversal of earth's magnetic field, with increased cosmic radiation levels and thus more skin cancers and miscarriages	unknown
▪ atmospheric oxygen depletion due to forest reduction and exaggerated fuel consumption	> 1000
▪ upcoming ice age	ca. 50 000
▪ possible collision with interstellar gas cloud assumed to be crossed by the earth every 60 million years, causing mass extinctions	ca. 50 000
▪ gamma ray burst from within our own galaxy, causing radiation damage to many living beings	between 0 and $5 \cdot 10^6$
▪ asteroid hitting the earth, generating tsunamis, storms, darkening sunlight, etc.	between 0 and $50 \cdot 10^6$
▪ neighbouring star approaching, starting comet shower through destabilization of Oort cloud and thus risk for life on earth	$> 10^6$
▪ instability of solar system	$> 100 \cdot 10^6$
▪ low atmospheric CO_2, content stops photosynthesis	$> 100 \cdot 10^6$
▪ ocean level increase due to earth rotation slowing/stop	$> 10^9$
▪ temperature rise/fall (depending on location) due to earth rotation stop	$> 10^9$
▪ sun runs out of fuel, becomes red giant, engulfs earth	$5.0 \cdot 10^9$
▪ sun stops burning, becomes white dwarf	$5.2 \cdot 10^9$
▪ earth core solidifies, removing magnetic field and thus earth's cosmic radiation shield	$10.0 \cdot 10^9$
▪ nearby nova (e.g. Betelgeuse) bathes earth in annihilation radiation	unknown
▪ nearby supernova (e.g. Eta Carinae) blasts over solar system	unknown
▪ galaxy centre destabilizes rest of galaxy	unknown
▪ universe recollapses – if ever (see page 200 ff.)	$> 20 \cdot 10^9$
▪ matter decays into radiation – if ever (see Appendix C)	$> 10^{33}$

* The web pages around cfa-www.harvard.edu/iau/lists/Closest.html provide more information on such events.

Situation	time scale in years from now
▪ problems with naked singularities	unknown, controversial
▪ the vacuum becomes unstable	unknown, controversial

Nevertheless, we leave aside these literally tremendous issues and continue in our adventure.

The essence of classical physics

We can summarize classical physics with a simple statement: *classical physics is the description of nature using the concept of infinity*. All the descriptions of nature used so for, be they for motion, space, time or observables, assume that the infinitely small and the infinitely large exist. Special relativity, despite the speed limit, still allows infinite proper velocity; general relativity, despite its black hole limit, still allows to approach it as much as possible. Mathematically, both integrals and derivatives are abbreviations of an infinite number of intermediate steps.

However, this approach does not completely convince. Some results, such as the atomic structure of matter, make us question the existence of the infinitely small.

Why is our mountain ascent not finished yet?

No problem is so formidable that
you can't walk away from it.
Charles Schultz

At the end of the 19th century, both Albert Michelson and Oliver Lodge – two well-known, mainly experimental physicists working on electrodynamics – claimed that electrodynamics and Galilean physics implied that the major laws of physics were well known. Their statements are often quoted as examples of flawed predictions, especially since their very own experiments lead to the development of relativity, which they failed to anticipate.

But these victorian physicists overlooked another contradiction between electrodynamics and nature for which they have no excuse. In our walk so far we found that clocks and meter bars are necessarily based on matter and electromagnetism. But as we just saw, we do not understand the stability of matter yet. Matter is made of small particles, but the relation between these particles and electricity is not clear. This implies that we do not yet understand space and time, since both are defined with measurement devices made of matter. It is also not clear whether infinitely small quantities really exist. There is a challenge waiting, namely the second part of our mountain ascent. The prize is to understand interactions.

Only the study of interactions allows to settle a further question the 19th century overlooked: if motion is *conserved* in collisions, what exactly is *exchanged* between colliding bodies? The fascinating path towards the answer is almost purely a sequence of surprises.

Subsequently, we need to rethink electromagnetism, as well as the other interactions we will discover, in the presence of space-time curvature. This challenge forms the third and final part of our mountain ascent. There the adventure becomes truly mind boggling and almost incredible. The reason is simple: both remaining parts of our mountain ascent require an approach for the description of motion which we have not encountered yet: quantum theory.

Finally, we still have not resolved the issue we mentioned at the end of Galilean physics: we still are defining space-time with help of objects, and objects with help of space-time. That will be the high point of our ascent. To be well prepared, we first take a break.

